1. At a certain gas station, 40% of the customers use regular unleaded gas, 35% use extra unleaded gas, and 25% use premium unleaded gas. Of those customers using regular gas, only 30% fill their tanks. Of those customers using extra gas, only 60% fill their tanks, while of those customers using premium gas, 50% fill their tanks.

   (a) (5%) What is the probability that the next customer will request extra unleaded gas and fill the tank?

   (b) (5%) What is the probability that the next customer fills the tank?

   (c) (10%) If the next customer fills the tank, what is the probability that regular gas is requested? What is the probability for Extra gas?

2. Find $\Pr(X > Y)$ assuming that $X$ and $Y$ are independent and that

   (a) (10%) $X$ and $Y$ are uniformly distributed between 0 and 1;

   (b) (10%) $X$ and $Y$ are geometrically distributed with parameters $p_X$ and $p_Y$, respectively. (That is, $\Pr(X = n) = p_X(1 - p_X)^{n-1}$.)

3. Let $X_1$, $X_2$ and $X_3$ be mutually independent random variables with Poisson distributions having means 1, 2, 3, respectively. That is,

$$\Pr(X_i = n) = \frac{e^{-i}i^n}{n!}$$

   for $i = 1, 2, 3$.

   (a) (10%) Find the moment-generating function of the sum $X_1 + X_2 + X_3$.

   (b) (10%) How is the sum $X_1 + X_2 + X_3$ distributed?

4. (20%) Solve the following system of difference equations

$$\begin{aligned} x_1(n+1) - x_1(n) - x_2(n) &= n \\ x_2(n+1) - x_1(n) + x_2(n) &= 3, \end{aligned}$$

   subject to the initial conditions $x_1(0) = 0$ and $x_2(0) = -10$.

5. (10%) Ten boys are grouped into five tennis doubles pairs. In how many ways can this be done?

6. (10%) How many terms are there in the expansion of $(a + b + c + d)^5$?