1. Suppose that $h(t)$ is the impulse response of a linear time-invariant (LTI) system given by

$$h(t) = e^{-t}u(t)$$

where $u(t)$ is the unit step function defined as follows:

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0. \end{cases}$$

   (a) (4%) Is the LTI system causal, stable? Why?

   (b) (4%) Find the output signal $y(t)$ as the input signal to the system is given by

$$x(t) = 1 + \frac{1}{2}\cos(2\pi t).$$

   (c) (7%) Assume that the input signal $x(t)$ is a real zero-mean white noise of power spectral density $N_0/2$. Find the autocorrelation function, the power spectral density, and the average power of $y(t)$.

2. Consider a first-order phase-locked loop (PLL) described by the following differential equation:

$$\frac{d\phi_e(t)}{dt} + 2\pi K_0 \sin[\phi_e(t)] = \frac{d\phi_1(t)}{dt}$$

where $\phi_1(t)$ is the phase to be tracked, $\phi_e(t)$ is the phase error, and K_0 is a constant called the loop-gain parameter.

   (a) (7%) We first examine the dynamic behavior of the PLL. Assume that originally $\phi_e = 0$ and then a unit step of magnitude $2\pi\Delta f$ is applied as $d\phi_1/dt$, i.e., $d\phi_1/dt = 2\pi\Delta f$, where $\Delta f < K_0$. Please use phase-plane plot ($d\phi_e/dt$ v.s. ϕ_e plot) to illustrate the operation of this PLL. Please find the stable equilibrium point and explain why it is.

   (b) (8%) This PLL can be used as an FM demodulator. Assume

$$\phi_1(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

   where $m(t)$ is the modulating message signal and k_f is a constant. The output is given by

$$v(t) = \frac{K_0}{k_v}\phi_e(t)$$

   where k_v is a constant. Suppose the following approximation is employed to linearize the differential equation:

$$\sin[\phi_e(t)] \simeq \phi_e(t)$$

   when $\phi_e(t)$ is small. Please use Fourier transform to find the steady-state response of $v(t)$. Also assume for the frequency range concerned, the loop gain K_0 is much larger than $|f|$, i.e., $K_0 \gg |f|$. Explain how demodulation can be achieved.

3. Consider the noisy model of a single-sideband modulation receiver shown below, with the modulated wave given by

$$s(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) - \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)$$

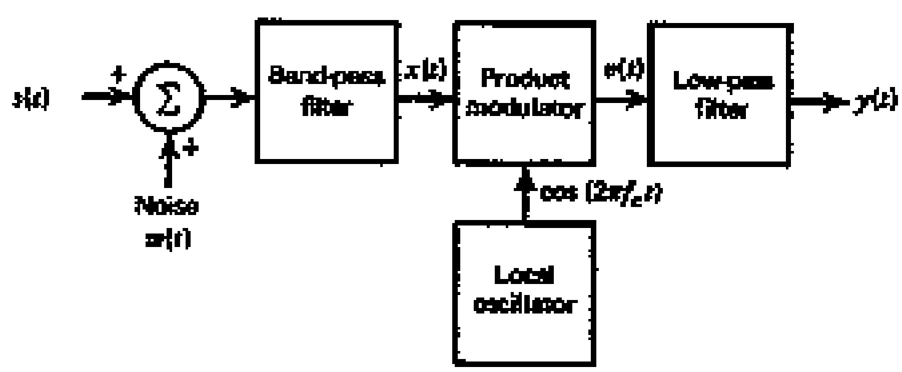
where $m(t)$ is the message signal of bandwidth W and average power P, $\hat{m}(t)$ is the Hilbert transform of $m(t)$, and f_c is the carrier frequency. The additive noise $w(t)$ is white Gaussian with zero mean and power spectral density $N_0/2$. The band-pass filter is assumed to be ideal with bandwidth the same as that of $s(t)$. Assume the recovered carrier $\cos(2\pi f_c t)$ is in perfect synchronism in both frequency and phase with the original carrier. The low-pass filter is also assumed to be ideal with bandwidth the same as that of $m(t)$.

(a) (3%) Write an expression for $x(t)$.

(b) (3%) Find the output $y(t)$.

(c) (6%) For the noise part of the $y(t)$ obtained in (b), please find and plot its power spectral density. What is its average power?

(d) (3%) Evaluate the output signal-to-noise power ratio.



4. Consider a communication system where four analog signals $m_1(t)$, $m_2(t)$, $m_3(t)$, and $m_4(t)$ are to be transmitted on a time-division multiplexed basis over a common channel. Assume that the bandwidth of $m_1(t)$ is 6 kHz and those of the other three signals are 2 kHz each. Also let each of the signals be sampled at its Nyquist rate.

(a) (4%) Design and plot a multiplexing scheme with the minimum possible commutator speed (rotations/second). You need to indicate the commutator speed.

(b) (3%) If the commutator output is uniformly quantized into 1024 levels and binary coded, what is the output bit rate?

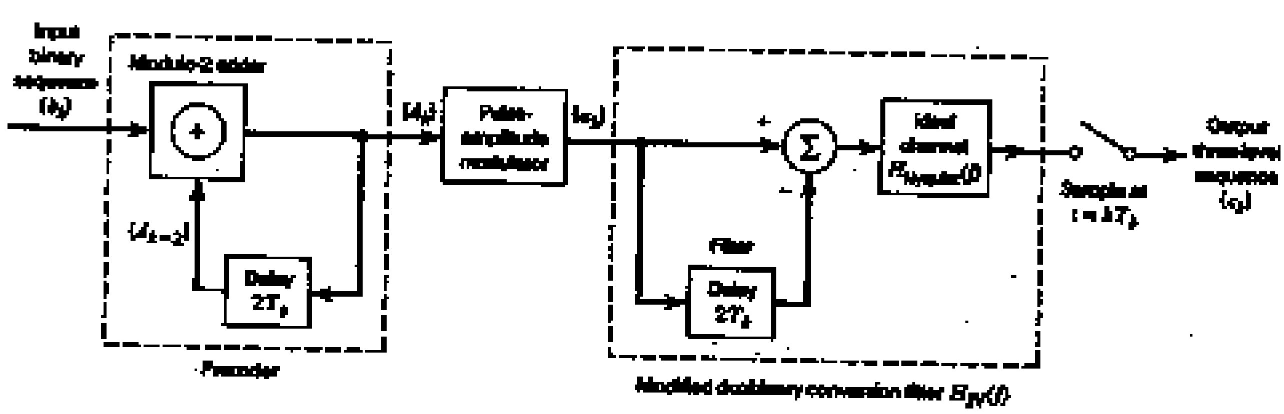
(c) (3%) Design an alternative multiplexing scheme with a higher commutator speed for the problem.

5. Consider a binary input sequence $\{b_k\}$ that consists of uncorrelated binary symbols 1 and 0, each having duration T_b. Assume that this sequence is applied to a modified duobinary signaling scheme as shown below, where the pulse amplitude modulator produces a two-level sequence of short pulses $\{a_k\}$ with $a_k = \pm 1$.

   (a) (5%) Does the received sequence $\{c_k\}$ have dc components? (You must justify your answer, or you will get no points.)

   (b) (5%) Derive a decision rule for detecting the original binary sequence $\{b_k\}$ from the received sequence $\{c_k\}$.

   (c) (5%) As compared to the binary pulse amplitude modulation systems, does the modified duobinary signaling scheme require a larger signal-to-noise ratio to achieve the same average probability of symbol error in the presence of noise? (You must justify your answer, or you will get no points.)



6. Consider a binary communication system that has one-dimensional signal vectors $s_1 = -\sqrt{E}$ and $s_2 = \sqrt{E}$. As shown below, the channel is characterized by additive Laplacian noise with density

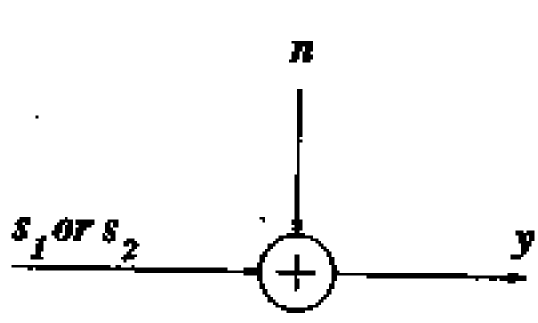
$$p(n) = \frac{1}{\sqrt{2}\sigma_n} \exp\left(-\sqrt{2}|n|/\sigma_n\right).$$

The a priori probabilities of the messages are $P(2) = 1 - P(1) = p$. The receiver compares the channel output $y = s_i + n$ to a threshold T, and chooses message "1" when $y < T$ and message "2" otherwise.

   (a) (6%) Derive an expression for the threshold T that minimizes the probability of error in terms of p and σ_n.

   (b) (3%) Express the signal-to-noise power ratio (SNR) in terms of E and σ_n.

   (c) (6%) Assume s_1 and s_2 occur with equal probability, i.e., $p = 1/2$. Express the probability of error as a function of SNR.



7. In a wireless communication system, one of two equally-likely data bits (-1 and 1) is transmitted with pulse-shaping function

$$p(t) = \begin{cases} 1, & 0 \leq t < T \\ 0, & \text{elsewhere.} \end{cases}$$

That is, for message "1", $s(t) = p(t)$ is transmitted, and for message "-1", $s(t) = -p(t)$ is transmitted. The received signal $y(t)$ is described by

$$y(t) = s(t) \star h(t) + n(t)$$

where $\star$ denotes convolution and the impulse response of the multi-path wireless channel is given by

$$h(t) = \delta(t) + \frac{1}{2}\delta\left(t - \frac{T}{2}\right)$$

in addition to the additive white Gaussian noise $n(t)$ of mean zero and power spectral density $N_0/2$.

(a) (5%) Draw the optimum receiver using matched filter(s).

(b) (5%) Find the average probability of error in terms of the Q function, where $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty \exp(-t^2/2)\, dt$.

(c) (5%) Repeat parts (a) and (b) if the wireless channel is an ideal additive white Gaussian noise channel with the same power spectral density and without multi-path, i.e., $h(t) = \delta(t)$. Compare your answers and explain the multi-path effect.