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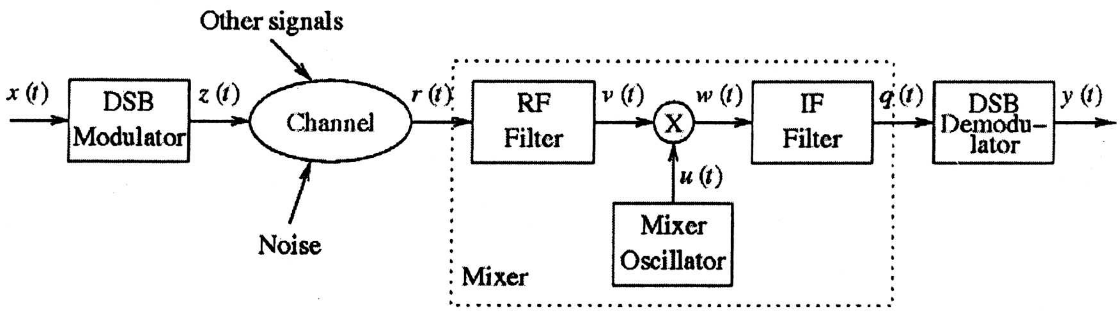
Problems:

1. (Total = 30%) Consider the following transmission system, where DSB stands for "double sideband," RF stands for "radio frequency" or "radiation frequency," and IF stands for "intermediate frequency":



參考用

(a) (5%) Let the frequency spectrum of the continuous-time signal $x(t)$ be given by $X(f)=\frac{1}{2000}\Pi\left(\frac{f}{2000}\right)$ where $\Pi(f)$ is the "unit pulse function," i.e., $\Pi(f)=1$ for $|f|\le 0.5$ and $\Pi(f)=0$ otherwise. Find $x(t)$ and express it in terms of the sinusoidal function and t. Note: Do *not* express it *directly* in terms of the sinc function.

(b) (6%) Following part (a), the DSB modulator output is given by $z(t)=A_c[1+ax(t)]\cos(f_c t)$ where f_c = 800 kHz, a = 1, and A_c = 2. Sketch the frequency spectrum $Z(f)$ of $z(t)$. Label your plot clearly so that one can reconstruct $z(t)$ exactly from your plot of $Z(f)$ with no uncertainty.

(c) (5%) Following part (b), let the channel and the RF filter be such that $v(t) = z(t)$. Let the mixer oscillator output $u(t)=\cos(f_m t)$ where f_m = 900 kHz. Sketch the frequency spectrum $W(f)$ of $w(t)$. Label it fully as in the case for $Z(f)$ in part (b).

(d) (3%) Following part (c), consider the IF filter. Specify a suitable passband for it.

(e) (5%) Following part (d), let the DSB demodulator be an envelope detector consisting of an ideal diode, a capacitor, and a resistor. Sketch the envelope detector; indicate clearly where its input and output are. *In addition*, give a proper upper bound and a proper lower bound for the *RC* constant for the signal considered in this problem.

(f) (6%) Consider the envelope detector in your answer to part (e). Prove mathematically whether it (from $q(t)$ to $y(t)$) is a linear system.

2. (Total = 15%) Consider the 16-QAM signaling scheme where each modulated signal can be expressed by

$$s_i(t) = \sqrt{\tfrac{2}{T_s}}\cdot\left(A_i\cos 2\pi f_c t + B_i \sin 2\pi f_c t\right),\quad \text{for } 0\le t< T_s, 1\le i\le 16,$$

where A_i and B_i take values on $\{-3a,-a,+a,+3a\}$ with equal probability, T_s is the symbol interval, and $f_c=\frac{1}{T_s}$ is the carrier frequency. The signal is transmitted over an AWGN channel with the signal model $y(t)=s(t)+n(t)$ where $s(t)\in\{s_1(t),\cdots,s_{16}(t)\}$ and $n(t)$ is AWGN independent of $s(t)$ with power spectral density $N_0/2$.

(a) (3%) Find the transmission bit rate of the system and express the average symbol energy in terms of a.

(b) (3%) In digital communications, it is common to convert the continuous waveforms to

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equivalent discrete vectors with appropriate choice of basis functions in the vector space in which the inner product of any two waveforms $f(t)$ and $g(t)$ is defined as $\langle f(t), g(t)\rangle \triangleq \int_0^{T_s} f(t)g(t)dt$.

Suppose we choose $\phi_1(t) = \sqrt{2/T_s}\cos 2\pi f_c t$ as one basis function in the signal space of 16-QAM. Please find the other basis function $\phi_2(t)$ that is orthonormal to $\phi_1(t)$. Sketch the signal constellation using this basis.

(c) (4%) Suppose the 4-bit sequence 0000 is assigned to the symbol point $(-3a, -3a)$ and the 4-bit sequence 0001 is assigned to the symbol point $(-3a, -a)$. Please specify the 4-bit sequences to the remaining 14 symbol points using *Gray coding* on the constellation plot in Part (b).

(d) (5%) The optimum receiver for the communication system with 16-QAM consists of two parallel branches: the in-phase branch $Y_1 \triangleq \langle y(t), \phi_1(t)\rangle$ and the quadrature branch $Y_2 \triangleq \langle y(t), \phi_2(t)\rangle$, where the inner product is defined in Part (b). Please explain why the decoding of the optimum receiver can be carried out by considering these two branches *separately*.

3. (Total = 20%) Alice is sending a binary message X (X=+1 or X=−1 equally likely) to her new boyfriend Bob through an AWGN communication channel. However, Alice's ex-boyfriend Chuck is so jealous that he is jamming their communications by sending a binary interference Z ($Z=+A$ or $Z=-A$ equally likely) over the same frequency band. The signal Y that Bob is receiving can be modeled as $Y = X + Z + N$,

where N is AWGN with zero mean and variance σ^2, and X, Z and N are statistically independent. As Chuck is very upset, he is sending Z with very large A which is much greater than 1 and σ. Assume that Bob knows the value of A, but he doesn't know which one ($+A$ or $-A$) was sent.

(You may need this: the Q-function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-y^2/2}dy$.)

(a) (5%) Bob first employs a simple decision strategy by approximating $Z + N$ as Gaussian with zero mean and variance $A^2 + \sigma^2$. What is the optimum decoding strategy, in the sense of minimum probability of decision error, under the Gaussian approximation of $Z + N$? Please find the corresponding probability of error decision. (Express your answer in terms of the Q-function.)

(b) (5%) Actually the decoding strategy in Part (a) doesn't provide a good decoding quality. So, instead of treating Z+N as Gaussian, Bob tries the following new decoding strategy:

$$\begin{cases} \widehat{X} = +1, & \text{if } Y - A \geq 0 \text{ or } 0 \leq Y + A < A, \\ \widehat{X} = -1, & \text{if } Y + A < 0 \text{ or } -A < Y - A < 0, \end{cases}$$

where $\widehat{X}$ denotes the decoded result of X. Please find the probability of error decision for this new decoding strategy. (Express your answer in terms of the Q-function.)

(c) (5%) Please interpret the decoding strategy in Part (b). Explain why the decoding strategy in Part (b) performs better than that in Part (a).

(d) (5%) What is the best decoding strategy, in the sense of minimum probability of error decision, that Bob can come up with? Please provide details of your derivations.

4. (Total = 15%) A communication system is described by a discrete model as $y_n = x_n * h_n + w_n$ where n is the time index, y_n is the received sequence, x_n is the transmitted symbol

sequence, h_n is a length-L sequence representing the channel's impulse response, $*$ means convolution, and w_n is AWGN.

(a) (5%) The purpose of a zero-forcing equalizer is to achieve the zero-ISI condition by operating on the received sequence without considering the AWGN. Please formulate the math equations that may lead to the design of a linear filter as a zero-forcing equalizer.

(b) (5%) The purpose of an MMSE equalizer is to minimize the Mean Squared Error (MSE) between the actual transmit symbol and the equalized symbol. Please give a mathematical formulation for the MSE of a linear equalizer.

(c) (5%) Assume the channel's impulse response to be $h_n = [1, -0.5]$. Find the linear zero-forcing equalizer for it. Please express the equalizer in its impulse response.

5. (Total = 20%) A speech signal is properly sampled with a sampling rate of 8 kHz. Then each sample is quantized with $q = 2^n$ uniform quantization levels in which n is the word length.

(a) (5%) Determine the minimum n such that the quantization noise is within $\pm 0.25\%$ of the peak-to-peak full-scale value.

(b) (5%) Following part (a), the quantized speech signal (in binary bits) is to be transmitted over a QPSK system. Determine the minimum symbol rate of the QPSK system such that the quantized speech data can be transmitted in real time.

(c) (5%) Assume that the quantized speech data first goes through a compression operation such that the data rate is only 20% of the original rate. Does the compressed data stream have lower entropy per data bit than the original data stream has? You need to give detailed explanations.

(d) (5%) Furthermore, a rate 2/3 convolutional code is added to enhance the reliability of the transmission. Now determine the minimum symbol rate of the QPSK system to support the real-time transmission of the encoded and compressed speech data.