台灣聯合大學系統 106 學年度碩士班招生考試試題

類組:電機類 科目:工程數學 B(3004)

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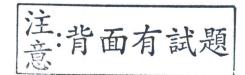
※請在答案卷內作答

Note: Detailed derivations are required to obtain a full score for Problem 2 to Problem 10.

- 1. (15%) Among the 10 statements below, only 5 are true and the other 5 are false. Find out which 5 are true. (You are not obligated to give explanations, but will get zero point if listing more than 5 of them).
 - (a) Let V be a vector space and $S \subseteq V$ be a subset. Then, $\operatorname{span}(S)$ is the intersection of all subspaces of V that contain S.
 - (b) Let $T : V \to W$ be a linear transformation. Let $S = \{v_1, v_2, ..., v_n\}$ be a subset of V. If S is linearly dependent, its image T(S) is also linearly dependent.
 - (c) The basis of any vector space uniquely exists.
 - (d) Let $T: V \to W$ be a linear transformation. If T is invertible, then $\dim(V) = \dim(W)$.
 - (e) Let $A \in M_{m \times n}(\mathbb{R})$ be an arbitrary matrix. If m < n, then $rank(A) > rank(A^t)$.
 - (f) Assume that $A \in M_{m \times n}(\mathbb{R})$ and $b \in M_{m \times 1}(\mathbb{R})$. Let x_1 and x_2 be two column vectors in \mathbb{R}^n . If $x_1 \neq x_2$ and $Ax_1 = b = Ax_2$, then the system of linear equations Ax = b has infinitely many solutions.
 - (g) Let A and B be square matrices of the same size. If AB = O, then $R(L_B) \supseteq N(L_A)$. (Remarks: L_A and L_B denote the linear transformation of matrix multiplication from the left.)
 - (h) Let A and B be square matrices of the same size. If AB = A, then B = I.
 - (i) Let A be a square matrix and $r \in \mathbb{R}$. Then, $\det(rA) = r \det(A)$.
 - (j) Assume that $A \in M_{3\times 3}(\mathbb{C})$ and $A^t A = -I$. Then, the entries in A cannot all be real numbers.
- **2.** (10%) Define a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ by T((1,0,0)) = (0,1,0), T((0,1,0)) = (0,0,1), and T((0,0,1)) = (1,0,0).
 - (a) (5%) Find a vector $u = (u_x, u_y, u_z)$ such that T(u) = u and $\sqrt{u_x^2 + u_y^2 + u_z^2} = 1$.
 - (b) (5%) Is $T : \mathbb{R}^3 \to \mathbb{R}^3$ one-to-one and onto? Why or why not?
- 3. (15%) Let W_1, \ldots, W_k be subspaces of a vector space V. The **direct sum** V of W_1, \ldots, W_k is defined if the following two conditions hold.

$$V = \sum_{i=1}^{n} W_i$$
 and $W_j \cap \sum_{i \neq j} W_i = \{0\} \ \forall j \ (1 \leq j \leq k)$

If the two conditions hold, then V is denoted by $V = W_1 \oplus \ldots \oplus W_k$. Prove or disprove (by providing a counterexample) of the following statements.





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- (a) (7%) If $V = W_1 \oplus \ldots \oplus W_k$. Then, for any distinct i and j, W_i and W_j intersect at exactly the zero vector.
- (b) (8%) If $V = \sum_{i=1}^{n} W_i$, and W_i and W_j intersect at exactly the zero vector for any distinct $i, j \ (1 \le i, j \le k)$. Then V is the direct sum of W_1, \ldots, W_k .
- 4. (10%) Let V be a finite-dimensional complex inner product space and T: V → V be a linear operator. T is normal if and only if TT* = T*T, where T* is the adjoint of T. Moreover, T is nilpotent if there exists n ∈ N such that Tⁿ is the zero operator. Prove the following statement. If T is both normal and nilpotent, then T is the zero operator itself.
- 5. (7%) Prove that if Θ is a random variable from the interval $[0, 2\pi]$, then the dependent variables $X = \sin \Theta$ and $Y = \cos \Theta$ are uncorrelated.
- **6.** (8%) Let X be a random variable; show that for $\alpha > 1$ and t > 0, $P(X \ge \frac{\ln \alpha}{t}) \le \frac{M_X(t)}{\alpha}$, where $M_X(t)$ is the moment generating function of X.
- 7. (10%) First a point Y is selected at random from the interval (0,1). Then another point X is selected at random from the interval (Y,1). Find the probability density function of X.
- **8.** (7%) A coin is tossed twice. Alice claims that the event of two heads is at least as likely if we know that the first toss is a head than if we know that at least one of the tosses is a head.
 - (a) (4%) Is she correct?
 - (b) (3%) Does it matter if it is a fair coin or an unfair coin? Compute the exact probabilities for each of the scenarios described above given that we know the coin is a fair coin.
- 9. (8%) Consider four independent rolls of a 6-sided die. Let X be the number of 1s and let Y be the number of 2s obtained. What is the joint PMF of X and Y?
- 10. (10%) Alice passes through four traffic lights on her way to work, and each light is equally-likely to be green or red, independent of the others.
 - (a) (5%) What is the mean and the variance of the number of red lights that Alice encounters?
 - (b) (5%) Suppose that each red light delays Alice by exactly two minutes. What is the variance of the time by which Alice is delayed by the red lights?

注:背面有試題

