注意：考試開始鈴響前，不得翻閱試題，

並不得書寫、畫記、作答。

國立清華大學 108 學年度碩士班考試入學試題

系所班組別：**動力機械工程學系 乙組**

考試科目(代碼)：**控制系統(1202)**

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

Q1. Figure 1 shows a rotor supported by two magnetic bearings. Bearing 1 is located at distance ℓ_1 from the center of mass of the rotor and generates a force f_1 to support the rotor, and bearing 2 is located at distance ℓ_2 from the center of mass of the rotor and generates a force f_2 to support the rotor. Denote the displacement of the center of mass of the rotor as x_c and the rotation angle of the rotor as θ. The mass of the rotor is m, and the moment of the inertia is I. Ignore the gravity and assume that the rotation angle θ is small so that the moment arm from bearing force f_1 to the center of mass is ℓ_1, and the moment arm from bearing force f_2 to the center of mass is ℓ_2.

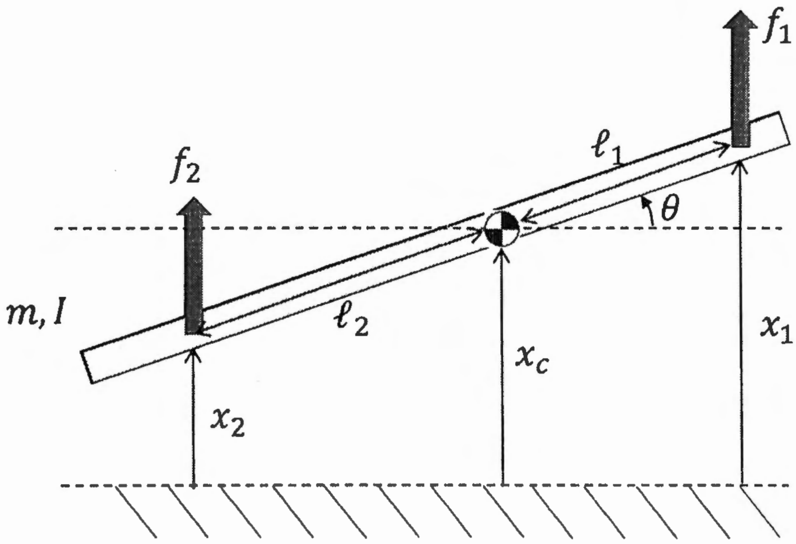

Figure 1

(a) Using Newton's law, derive the differential equations to describe the dynamics of x_c and θ. **(2 pts)** Let $\mathcal{L}\{x_c(t)\} = X_c(s)$, $\mathcal{L}\{\theta(t)\} = \Theta(s)$, $\mathcal{L}\{f_1(t)\} = F_1(s)$, and $\mathcal{L}\{f_2(t)\} = F_2(s)$. From the differential equations you derived, it can be shown that

$$\begin{bmatrix} X_c(s) \\ \Theta(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix}.$$

Compute $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$, and $G_{22}(s)$. **(4 pts)**

(b) Because of the electro-magnetic interaction, the magnetic forces can be expressed as $f_1 = k_x x_1 + k_i i_1$, and $f_2 = k_x x_2 + k_i i_2$, where $k_x > 0$ and $k_i > 0$ are electromagnetic coefficients, $x_1(x_2)$ is the rotor's displacement at the bearing 1 (2) (See Figure 1), and $i_1(i_2)$ is the control input current to bearing 1 (2). Assume θ is small, derive x_1 as a linear combination of x_c and θ, and x_2 as a linear combination of x_c and θ. **(2 pts)** Under the assumption that $I = m\ell_1\ell_2$, Compute $P_{11}(s)$, $P_{12}(s)$, $P_{21}(s)$, and $P_{22}(s)$ in the system

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}. \quad \textbf{(12 pts)}$$

(c) Using the transfer functions you find in part (b), compute the open-loop poles of the system. Is the system stable? **(5 pts)**

Q2. Consider a control system in Figure 2 in which $P(s) = \frac{1}{s^2-1}$.

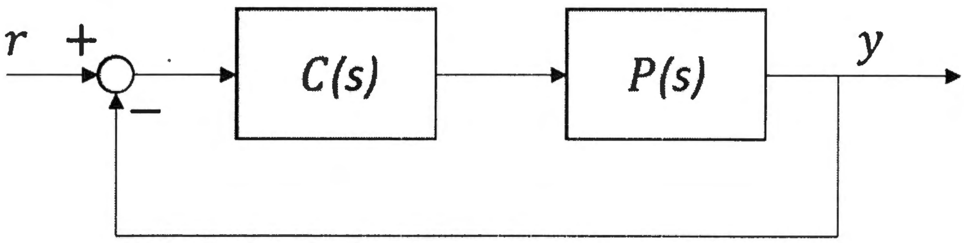

Figure 2

(a) Plot the Bode plot of $P(s)$. **(4 pts)**

(b) The target is to stabilize $P(s)$ using a lead compensator $C(s) = K\frac{(1+aTs)}{(1+Ts)}$ in which $K, a, T > 0$. Assuming that the steady-state error due to a step reference input should be limited to be within 4%, what is the minimum value for K? **(3 pts)** Under the choice of the minimum K, it is desired that the phase margin is 45°. Determine the parameters a and T. What is the crossover frequency you achieve? You may want to conduct the compensator design using the Bode plot in (a). **(13 pts)**

(c) Is it possible to achieve the same design specifications using a lag compensator? Show your reasoning. **(5 pts)**

Q3.

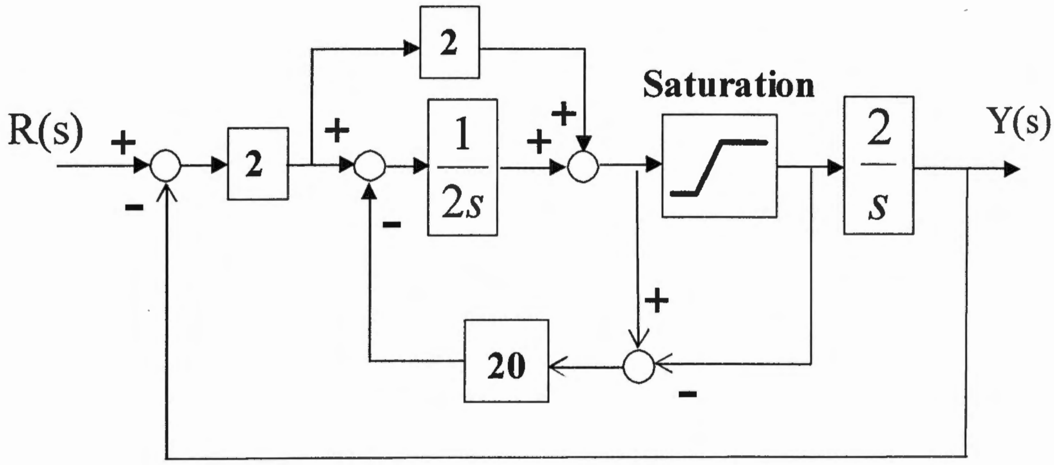

Figure 3

The block diagram above shows an example of anti-windup for integral control system. If we consider the saturation element as an equivalent gain, the root locus can be drawn and a prediction of the response to large inputs can be made.

(a) Compute the roots of the closed loop characteristic equation if the saturation block has a gain of 1 **(5 pts)**

(b) Replace the saturation block with an equivalent gain K (the slope is K before saturation). Sketch the root locus with respect to K (K>0). **(10 pts)**

(c) For (b), calculate and give the break-away points and break-in points if there is any. **(5 pts)**

(d) Based on looking at the root locus, what is the largest value of slope K at the saturation block to result in a stable closed-loop system? **(5 pts)**

Q4.

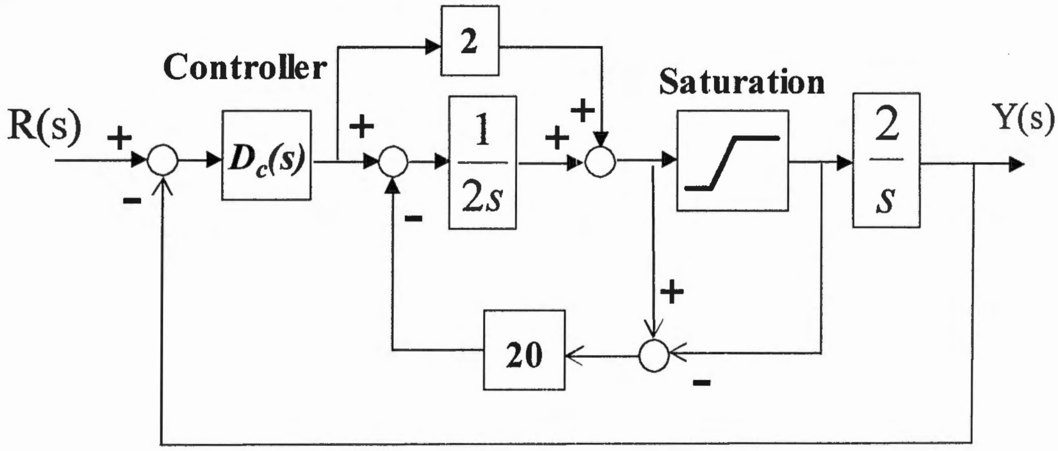

Figure 4

Ziegler-Nichols Tuning for the Regulator $D_c(s) = k_P(1 + 1/T_I s + T_D s)$, Based on the Ultimate Sensitivity Method

Type of Controller	Optimum Gain
P	$k_P = 0.5K_u$
PI	$\begin{cases} k_P = 0.45K_u \\ T_I = \frac{P_u}{1.2} \end{cases}$
PID	$\begin{cases} k_P = 0.6K_u \\ T_I = 0.5P_u \\ T_D = 0.125P_u \end{cases}$

Table 1 Ziegler-Nichols Tuning method

(a) In the case, the slope of the saturation element is **2** and the saturation signal is too large to be reached. Based on the Ziegler-Nichols Tuning method shown as above table, design a **PI** controller for $D_c(s)$ shown in above block diagram. Give the values of your (k_p, T_I) **(10 pts)**

(b) Determine the **relevant error constant** with respect to reference input R(s) for this closed-loop system with your PI controller $D_c(s)$ **(5 pts)**

Q5. Given the state space representation of the plant.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 50 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Design a state feedback controller such that the closed-loop response to a unit-step input has

$$M_p = 4.33\%, \quad t_s = 0.0474 \sec$$

(10 pts)