1. Suppose that $f(x,y,t) = x^2 y - e^{2y} + t^2$. Which of the following equations is correct?

(a) If $x = x(t)$ and $y = y(t)$, $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

(b) $\frac{\partial f}{\partial x} = xy$ (c) $\frac{\partial f}{\partial y} = x^2 - 2e^{2y}$ (d) $\frac{\partial f}{\partial t} = t$ (e) none of the above

2. Suppose that $x = x(u,v),\ y = y(u,v)$. Which of the following equations is correct?

(a) $\frac{\partial x}{\partial u}\cdot\frac{\partial u}{\partial x} = 1$ (b) $\frac{\partial x}{\partial u}\cdot\frac{\partial v}{\partial y} = 1$ (c) $\frac{\partial(x,y)}{\partial(u,v)} = \frac{\partial(u,v)}{\partial(x,y)}$

(d) $\frac{\partial x}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial x}{\partial v}\cdot\frac{\partial v}{\partial x} = 1$ (e) $\frac{\partial x}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial x}{\partial v}\cdot\frac{\partial v}{\partial y} = 1$

3. If $xy - y^3 = 1$, $y'(x)$ at $x = 0$ is equal to

(a) -1 (b) $-\frac{1}{2}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{4}$ (e) $-\frac{1}{5}$

4. The integral $\int_0^{\pi}\int_{y/2}^{\pi/2} \frac{\sin x}{x}\,dxdy$ is equal to

(a) 2 (b) 1 (c) 1/2 (d) π (e) $\pi/2$

5. The polar coordinates (r,θ) and the Cartesian coordinates (x,y) are related by $x = r\cos\theta,\ y = r\sin\theta$. Which of the following equations for the unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}$ is INCORRECT?

(a) $\hat{\mathbf{r}} = \cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}$ (b) $\hat{\boldsymbol{\theta}} = -\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}$ (c) $\frac{d\hat{\mathbf{r}}}{d\theta} = \hat{\boldsymbol{\theta}}$

(d) $\frac{d\hat{\boldsymbol{\theta}}}{d\theta} = -\hat{\mathbf{r}}$ (e) $\frac{d\hat{\mathbf{r}}}{dr} = \hat{\boldsymbol{\theta}}$

6. The spherical coordinates (ρ, ϕ, θ) of a point P are defined by that ρ is the distance form origin O to P, ϕ the angle between the z axis and OP, θ the angle between the x axis and the projection of OP on the x-y plane. The Jacobian $\dfrac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)}$ is equal to

(a) $\rho^2\cos\phi$ (b) $\rho^2\sin\phi$ (c) $\rho^2\cos\phi\sin\theta$ (d) $\rho^2\cos\phi\cos\theta$ (e) $\rho^2\sin\phi\cos\theta$

7. If Φ is a scalar field and $\mathbf{A},\mathbf{B}$ are vector fields, which of the following equations is INCORRECT?

(a) $\nabla\cdot(\Phi\mathbf{A}) = \nabla\Phi\cdot\mathbf{A} + \Phi\nabla\cdot\mathbf{A}$ (b) $\nabla\times(\Phi\mathbf{A}) = \nabla\Phi\times\mathbf{A} + \Phi\nabla\times\mathbf{A}$

(c) $\nabla\cdot(\nabla\times\mathbf{A}) = 0$ (d) $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$ (e) none of the above

8. A vector field $\mathbf{A}(x,y,z) = xy\,\hat{\mathbf{i}} + yz\,\hat{\mathbf{j}} - xy^2\,\hat{\mathbf{k}}$, then $\nabla\cdot\mathbf{A}$ is equal to

(a) $x+y$ (b) $y+z$ (c) $y+z-2xy$ (d) $xy+yz-xy^2$ (e) none of the above

9. Suppose that $\mathbf{V}(x,y) = 2xy\,\hat{\mathbf{i}} + x^2\,\hat{\mathbf{j}}$ is a gradient field and a curve $\mathbf{R}(\tau) = (1+\cos\tau)\hat{\mathbf{i}} + \sin\tau\,\hat{\mathbf{j}}$ is defined by $-\pi/2 \le \tau \le \pi/2$. The line integral ${}_{=-\pi/2}^{=\pi/2}\ \mathbf{V}\cdot d\mathbf{R}$ is equal to

(a) 2 (b) 1 (c) 0 (d) −1 (e) −2

10. A function $f(x)$ defined in $0 < x < L$ has a Fourier half range cosine expansion $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\dfrac{n\pi x}{L}$. Which of the following equations is correct?

(a) $a_n = \dfrac{1}{L}\int_0^L f(x)\cos\dfrac{n\pi x}{L}dx$ (b) $a_n = \dfrac{2}{L}\int_0^L f(x)\cos\dfrac{n\pi x}{L}dx$

(c) $a_n = \dfrac{1}{L}\int_0^L f(x)\cos\dfrac{2n\pi x}{L}dx$ (d) $a_n = \dfrac{2}{L}\int_0^L f(x)\cos\dfrac{2n\pi x}{L}dx$ (e) none of the above

11. The Sturm-Liouville equation has the form $[p(x)y']' + q(x)y + \lambda w(x)y = 0,\ a < x < b$. For the differential equation $y'' - 2y' + \lambda y = 0,\ 0 < x < \pi$, which of the following function, when multiplied into the above equation, can turn it into a Sturm-Liouville equation.

(a) x (b) $-x$ (c) e^x (d) e^{-x} (e) e^{-2x}

12. The Fourier transform of $f(x)$ is defined by $F\{f(x)\} = \int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx$. If $f^{(n)}(x)$ is the nth derivative, $F\{f^{(n)}(x)\}$ is equal to

(a) $\omega^n F\{f(x)\}$ (b) $(i\omega)^n F\{f(x)\}$ (c) $(-\omega)^n F\{f(x)\}$

(d) $(-i\omega)^n F\{f(x)\}$ (e) none of the above

13. If $H(x)$ is the Heaviside step function and $f(x) = H(x+1) - H(x-1)$. The Fourier transform $F\{f(x)\}$ is equal to

(a) $2\dfrac{\sin\omega}{\omega}$ (b) $2\dfrac{\cos\omega}{\omega}$ (c) $\dfrac{\sin\omega}{\omega}$ (d) $\dfrac{\cos\omega}{\omega}$ (e) none of the above

14. Consider the wave equation $c^2u_{xx} = u_{tt}, u(0,t) = u(L,t) = 0, u(x,0) = f(x), u_t(x,0) = g(x)$ for $0 \le x \le L, 0 \le t$. By the method of separation of variables, we have $u(x,t) = X(x)T(t)$ and $\dfrac{X''}{X} = \dfrac{T''}{c^2T} = \text{constant} = -\kappa^2$, $\kappa > 0$. If $n = 1,2,3\ldots$, κ is equal to

(a) $\dfrac{(2n-1)\pi}{L}$ (b) $\dfrac{(2n-1)\pi}{2L}$ (c) $\dfrac{2n\pi}{L}$ (d) $\dfrac{n\pi}{L}$ (e) $\dfrac{n\pi}{2L}$

15. Following the above problem, $u(x,t)$ is the superposition of the possible $X(x)T(t)$ and has the form $u(x,t) = \sum_{n=1}^{\infty} A_n X_n(x)T_n(t)$. Then $X_n(x)$ is equal to

(a) $\sin\dfrac{(2n-1)\pi}{L}x$ (b) $\cos\dfrac{(2n-1)\pi}{2L}x$ (c) $\sin\dfrac{2n\pi}{L}x$ (d) $\sin\dfrac{n\pi}{L}x$ (e) $\cos\dfrac{n\pi}{2L}x$

16. Which of the following complex functions is NOT differentiable?

(a) $\sin z$ (b) z (c) e^z (d) $\dfrac{1}{z}$ $(z \ne 0)$ (e) $|z|^2$

17. Which of the following is a particular solution $x_p(t)$ for the differential equation $x'' + 2x' + 3x = \cos 3t$?

(a) $\mathrm{Re}[-\frac{\ }{6(1-i)}]$ (b) $\mathrm{Re}[-\frac{e^{i3t}}{6(1+i)}]$ (c) $\mathrm{Re}[-\frac{e^{i3t}}{3(1-i)}]$ (d) $\mathrm{Re}[\frac{e^{i3t}}{3(1+i)}]$ (e) $\mathrm{Re}[-\frac{e^{i3t}}{3(1+i)}]$

18. The complex function $f(z) = \frac{z-1}{z(z+2)}$ can NOT be expanded into a unique Laurent series about $z = i$ in the region

(a) $|z-i| < \frac{1}{2}$ (b) $|z-i| < 1$ (c) $|z-i| < \sqrt{5}$ (d) $|z-i| > \sqrt{5}$ (e) none of the above

19. On the complex z plane, let C be a closed circle centered at $z = 1$ with a radius of 2 and oriented counterclockwis. Then $\oint_c \frac{dz}{z^2 - 2}$ is equal to

(a) 0 (b) $\frac{\pi}{\sqrt{2}} i$ (c) $\sqrt{2}\pi i$ πi (d) $-\frac{\pi}{\sqrt{2}} i$ (e) $-\sqrt{2}\pi i$

20. On the complex z plane, let C be the square contour $0 \to 2 \to 2+2i \to 2i \to 0$. The integral $\oint_c \frac{1}{z^2 - 2iz - 1} dz$ is equal to

(a) 0 (b) πi (c) $2\pi i$ (d) $-\pi i$ (e) $-2\pi i$

21. The general solution of $y' = \frac{y}{x} + 3\sqrt{\frac{x}{y}}$ is (a) $y = x(\frac{9}{2}\ln x + c)^{2/3}$, (b) $y = x(\frac{9}{2}\ln x + c)^{-2/3}$, (c) $y = x(\frac{9}{2}\ln x + c)^{3/2}$, (d) $y = x(\frac{9}{2}\ln x + c)^{-3/2}$, (e) none of the above

22. The general solution of $y' - 4y = 4y^2$ is (a) $y = \frac{e^{-4x}}{c + e^{-4x}}$, (b) $y = \frac{e^{-4x}}{c - e^{-4x}}$, (c) $y = \frac{e^{4x}}{c + e^{4x}}$, (d) $y = \frac{e^{4x}}{c - e^{4x}}$, (e) $y = \frac{e^{4x}}{c + e^{-4x}}$

23. Find the general solution of the following equation $y'' + iy' - y = 0$

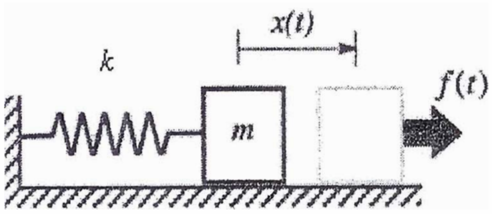
(a) $y(x) = e^{\frac{ix}{2}}(Ae^{\frac{\sqrt{2}x}{2}} + Be^{\frac{-\sqrt{2}x}{2}})$　(b) $y(x) = e^{-\frac{ix}{2}}(Ae^{\frac{\sqrt{2}x}{2}} + Be^{\frac{-\sqrt{2}x}{2}})$

(c) $y(x) = e^{\frac{ix}{2}}(Ae^{\frac{\sqrt{3}x}{2}} + Be^{\frac{-\sqrt{3}x}{2}})$　(d) $y(x) = e^{-\frac{ix}{2}}(Ae^{\frac{\sqrt{3}x}{2}} + Be^{\frac{-\sqrt{3}x}{2}})$　(e) none of the above

24. (*Forced oscillation) During the* disastrous *921 Earthquake*, a skyscraper suffered from earthquake-induced vibration. The system aforementioned can be simplified as the following figure: where ***m*** denotes the mass of the skyscraper, ***k*** is spring constant, and the time-varying force $f(t) = f_0 \cos\Omega t$ is applied due to the earthquake. Use Newton's second law to model this system as

(a) $mx'' + kx' = f_0 \cos\Omega t$　(b) $mx'' - kx' = f_0 \cos\Omega t$　(c) $mx'' + kx = f_0 \cos\Omega t$

(d) $mx'' - kx = f_0 \cos\Omega t$　(e) none of the above



25. Based on your answer in the previous question, determine the natural frequency ω of this building

(a) $\omega = 2\pi\sqrt{\frac{k}{m}}$　(b) $\omega = 2\pi\sqrt{\frac{m}{k}}$　(c) $\omega = \sqrt{\frac{m}{k}}$　(d) $\omega = \sqrt{\frac{k}{m}}$　(e) none of the above

26. The general solution of $y'' - 2y' + 5y = e^x \cos 2x$ is

(a) $y = e^x(A\cos 2x + B\sin 2x) - \frac{1}{4}xe^x \sin 2x$　(b) $y = e^x(A\cos 2x + B\sin 2x) + \frac{1}{4}xe^x \sin 2x$

(c) $y = e^x(A\cos 2x + B\sin 2x) - \frac{1}{4}x\sin 2x$　(d) $y = e^x(A\cos 2x + B\sin 2x) + \frac{1}{4}x\sin 2x$

(e) none of the above

27. The convergence radius of $\sum_{n=0}^{\infty} \frac{n^{50}}{n!}(x+7)^n$

(a)0 (b)1 (c)2 (d)3 (e) none of the above

28. The integral $\Gamma(x)=\int_0^{\infty} t^{x-1}e^{-t}dt$ (x>0) is termed the gamma function. Find $\Gamma(2)$

(a)-1 (b)0 (c)1 (d)2 (e)$\sqrt{\pi}$

29. Find $L\{(t^2+1)u(t-1)\}$, where *u(t)* is step function

(a) $e^{-s}(\frac{2}{s^3}+\frac{1}{s})$ (b) $-e^{-s}(\frac{2}{s^3}+\frac{1}{s})$ (c) $e^{-s}(\frac{2}{s^3}+\frac{2}{s^2})$

(d) $-e^{-s}(\frac{2}{s^3}+\frac{2}{s^2})$ (e) $e^{-s}(\frac{2}{s^3}+\frac{2}{s^2}+\frac{2}{s})$

30. Find $L^{-1}\{\ln\frac{s+a}{S+b}\}$

(a) $(e^{-bt}+e^{-at})$ (b) $(e^{-bt}-e^{-at})$ (c) $(\frac{e^{-bt}+e^{-at}}{t})$

(d) $(\frac{e^{-bt}-e^{-at}}{t})$ (e) $(\frac{e^{bt}+e^{at}}{t})$

31. Which of the following methods allows to calculate the numerical solution of initial-value problems with the least error (for the same interval)

(a)Euler method (b) Tangent-line method (c)Midpoint rule

(d)2nd-order Runge-Kutta method (e) 4th-order Runge-Kutta method

32. Classify the singularity at the origin for the following system: $\begin{cases} x' = x-4y \\ y' = x+y \end{cases}$

(a) center (b)focus (c)node (d)saddle (e) none of the above

33. The direction cosines of vector $\boldsymbol{u}=(u_1, u_2, u_3)$ in 3-space are defined as $l_1 \equiv \cos\alpha$, $l_2 \equiv \cos\beta$, $l_3 \equiv \cos\gamma$, where α, β, and γ are the angles between u and the positive coordinate. For the case of u=(2,-1,5), which of the following statement is ***Incorrect***?

(a) $l_1 = 2/\sqrt{30}$ (b) $l_2 = -1/\sqrt{30}$ (c) $l_3 = 5/\sqrt{30}$

(d) $\alpha+\beta+\gamma=180^0$ (e) $l_1^2+l_2^2+l_3^2=1$ for all kinds of vectors $\boldsymbol{u}$ vectors

34. Find the answer of the following linear system:

$$\begin{aligned} 6x_1 - 2x_2 - 4x_3 + x_4 &= 2 \\ 3x_1 - 3x_2 - 6x_3 + x_4 &= -4 \\ -12x_1 + 8x_2 + 21x_3 - 8x_4 &= 8 \\ -6x_1 - 10x_3 + 7x_4 &= -43 \end{aligned}$$

The incorrect number for the solution set is (a) $\frac{9}{2}$ (b)0 (c) $\frac{69}{10}$ (d) $\frac{-6}{5}$ (e)-4

(Hint: you can use either Gauss elimination or and LU Factorization)

35. The determinant of the following matrix $\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 3 & 1 & -1 \\ 0 & 4 & 5 & 0 \\ 1 & 2 & 3 & 6 \end{bmatrix}$ is (a)132, (b)-132, (c)200, (d)-200, (e) 100

(Hint: You can use co-factor expansion or/and triangularization to do so.)

36. For what value(s) of the λ parameters do(es) the following homogeneous system admit nontrivial solutions

$$\begin{aligned} x+y+z &= \lambda x \\ y+z &= \lambda y \\ 2z &= \lambda z \end{aligned}$$

(a)-2 (b)-1 (c)1 (d)2 (e)1 and 2

37. Use the Gram-Schmidt formula to obtain an orthonormal set from the given linear independent set (1, 0, 0), (1, 1, 0), (1, 1, 1). Which of the following vector(s) do(es) ***NOT*** belong to this orthonormal set?

(a) (1, 0, 0) (b) $(0, \sqrt{2}/2, \sqrt{2}/2)$ (c) (0, 0, 1) (d) (0, 1, 0) (e) (0, 0, 1) and (0, 1, 0)

38. The inverse matrix of $\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$ is

(a) $\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$ (b) $\begin{bmatrix} -\cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$ (c) $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & -1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$

(d) $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & -\cos\theta \end{bmatrix}$ (e) $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

39. The orthogonal basis of the following matrix $\begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$?

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(e) none of the above

40. Diagonalize the given matrix A= $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 3 \\ -1 & 3 & 4 \end{bmatrix}$. Which one is the corresponding diagonal matrix ?

(a) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (e) none of the above

41. When an ideal gas undergoes a reversible adiabatic process, which one in the following is correct? ($\gamma = C_p/C_v$, R is the gas constant)

(a) $PV^{\gamma-1}$ = constant (b) TV^{γ} = constant (c) PV^{R} = constant

(d) TV^{R} = constant (e) None of the above

42. When an ideal gas undergoes a reversible isothermal process from V_1 to V_2, which is the work done by the system?

(a) $RT\ln\frac{V_1}{V_2}$ (b) $RT\ln\frac{V_2}{V_1}$ (c) $RT\ln\frac{P_2}{P_1}$

(d) $RT\ln\frac{P_1V_2}{P_2V_1}$ (e) None of the above

43. In a closed system of constant temperature and constant volume, the criterion for equilibrium is:

(a) The Gibbs free energy (G) has its minimum value.
(b) The entropy (S) has its maximum value.
(c) The enthalpy (H) has its minimum value.
(d) The Helmholtz free energy (A) has its minimum value.
(e) None of the above.

44. In a closed system held at constant entropy and constant pressure, the criterion for equilibrium is:

(a)The Gibbs free energy (G) has its minimum value.
(b)The entropy (S) has its maximum value.
(c)The enthalpy (H) has its minimum value.
(d)The Helmholtz free energy (A) has its minimum value.
(e)None of the above.

45. The chemical potential of species *i* (μ_i) is equal to:

(a) $\left(\frac{\partial G}{\partial n_i}\right)_{T,V,n_j}$ (b) $\left(\frac{\partial U}{\partial n_i}\right)_{T,V,n_j}$ (c) $\left(\frac{\partial H}{\partial n_i}\right)_{T,V,n_j}$ (d) $\left(\frac{\partial A}{\partial n_i}\right)_{T,V,n_j}$ (e) none of the above

46. If Ω_{conf} is the number of spatial configuration of a system and Ω_{th} is the number of arrangement of particles among the energy levels, the total entropy of the system is:

(a) $k(\ln\Omega_{conf})/(\ln\Omega_{th})$ (b) $k(\ln\Omega_{conf})+k(\ln\Omega_{th})$ (c) $(k\ln\Omega_{conf})(\ln\Omega_{th})$

(d) $k\ln(\Omega_{conf}+\Omega_{th})$ (e) None of the above.

47. In a two-component (A and B) system, phase 1 and phase 2 coexist in equilibrium. Which description is correct? $\overline{G_i}$ is the partial molar Gibbs free energy of component *i*.

(a) $\overline{G_A}^{(1)}=\overline{G_B}^{(1)}$ (b) $\overline{G_A}^{(1)}=\overline{G_B}^{(2)}$ (c) $\overline{G_A}^{(1)}=\overline{G_A}^{(2)}$

(d) $\overline{G_A}^{(1)}+\overline{G_A}^{(2)}=\overline{G_B}^{(1)}+\overline{G_B}^{(2)}$ (e) None of the above.

48. Which relationship is incorrect?

(a) $\left(\frac{\partial U}{\partial S}\right)_{v,comp}=\left(\frac{\partial H}{\partial S}\right)_{P,comp}$ (b) $\left(\frac{\partial U}{\partial V}\right)_{S,comp}=\left(\frac{\partial A}{\partial V}\right)_{T,comp}$

(c) $\left(\frac{\partial H}{\partial P}\right)_{S,comp}=\left(\frac{\partial G}{\partial P}\right)_{T,comp}$ (d) $\left(\frac{\partial A}{\partial T}\right)_{V,comp}=\left(\frac{\partial G}{\partial T}\right)_{P,comp}$

(e) None of the above.

49. Which relationship is correct?

(a) $\left(\frac{\partial T}{\partial V}\right)_{S,comp}=-\left(\frac{\partial P}{\partial S}\right)_{V,comp}$ (b) $\left(\frac{\partial T}{\partial P}\right)_{S,comp}=-\left(\frac{\partial V}{\partial S}\right)_{P,comp}$

(c) $\left(\frac{\partial S}{\partial V}\right)_{T,comp}=-\left(\frac{\partial P}{\partial T}\right)_{V,comp}$ (d) $\left(\frac{\partial S}{\partial P}\right)_{T,comp}=\left(\frac{\partial V}{\partial T}\right)_{P,comp}$

(e) None of the above.

50 Which one is the van der Waals equation? (a and b are positive values.)

(a) $(P+\frac{a}{V^2})(V-b)=RT$ (b) $(P-\frac{a}{V^2})(V+b)=RT$

(c) $(P+\frac{a}{V^2})(V+b)=RT$ (d) $(P-\frac{a}{V^2})(V-b)=RT$

(e) None of the above.

51. The melting temperature of ice is 0^oC at 1 atm. When the pressure changes to 10 atm, the melting temperature of ice is:

(a) 0^oC (b) $<0^oC$ (c) $>0^oC$

(d) Depending on other parameters (e) None of the above.

52. When a three-component system is in equilibrium (pressure is fixed at 1 atm) with zero degree of freedom, there must coexist

(a) one phase (b) two phases (c) three phases

(d) four phases (e) None of the above.

53. One mole of an ideal gas A (at 1 atm) and one mole of an ideal gas B (at 2 atm) are mixed to form an ideal gas solution of final pressure of 1 atm. The change of Gibbs free energy due to mixing is given by

(a)-2 RT ln 2 (b) 2 RT ln 2 (c) -3RT ln 2

(d) -3RT ln 3 (e) None of the above.

54. At constant temperature, the change of molar Gibbs free energy (G) of a nonideal gas can be described as

(a) $dG = RTd\ln P$ (P is pressure) (b) $dG = RTd\ln V$(V is volume)

(c) $dG = VdP$ (d) $dG = P\,dV$ (e)None of above

55. In mixing of ideal gases, if P_i is the pressure of component *i* before mixing and p_i is the partial pressure of component *i* after mixing, n_i and X_i are mole number and mole fraction of component *i*, respectively, the entropy of mixing can be described by

(a) $-\sum_i n_i R\ln p_i$ (b) $-\sum_i n_i R\ln P_i$ (c) $-\sum_i n_i R\ln X_i$ (d) $-\sum_i X_i R\ln X_i$

(e) None of the above.

56. Which thermodynamic function has an absolute value?

(a) Internal energy (b) Enthalpy (c) Entropy

(d) Gibbs free energy (e) None of the above.

57. During a reversible isothermal compression of one mole ideal gas from (P_1,V_1) to (P_2,V_2), the change in the entropy of the gas is

(a) $R\ln(V_1/V_2)$ (b) $R\ln(V_2/V_1)$ (c) $RT\ln(P_1/P_2)$ (d) $RT\ln(V_1/V_2)$ (e) None of the above

58. Which statement is correct for ideal gases?

(a) $\left(\frac{\partial U}{\partial V}\right)_T = 0$ (b) $\left(\frac{\partial H}{\partial V}\right)_S = 0$ (c) $\left(\frac{\partial U}{\partial T}\right)_V = 0$ (d) $\left(\frac{\partial H}{\partial S}\right)_P = 0$

(e) None of the above

59. A two- component system held at 1 atm and a certain temperature may consist of different number of phases. Which one of the following can conclude the system in equilibrium:

(a) the system has one phase, (b) the system has two phase,
(c) the system has three phase, (d) the system has four phase,
(e) None of the above.

60. Which of the following is negative for ideal gases?

(a) $\left(\frac{\partial S}{\partial T}\right)_P$ (b) $\left(\frac{\partial H}{\partial T}\right)_P$ (c) $\left(\frac{\partial G}{\partial P}\right)_T$ (d) $\left(\frac{\partial S}{\partial P}\right)_T$ (e) None of the above.

<u>Questions for (61-63)</u>

A first order phase change occurs for element A at 450 K under equilibrium conditions. The high temperature form (β) may be supercooled in a metastable state to temperature below the transformation temperature. Atomic weight=20 P=1 atm; T_{trans}=450 K, ρ^{α}=5g/cc, ρ^{β} =4g/cc

ΔH_{trans}=500 Cal/mole (from α to β)

Cp^{α}=6 (Cp is valid for 250 K<T<500K, unit:cal/K-mole)

Cp^{β}=7 (Cp is valid for 250 K<T<500K, unit:cal/K-mole)

61. What is the ΔG (Gibbs free energy change)at 300 K for the phase transformation from β to α?
(ln 450=6.11, ln 300=5.7) (a) 500 (b) 306 (c)-217 (d)-806 (e) -1020 cal/mole.

62. The vapor pressure of the α phase is 3×10^{-5} atm at 300 K. What is the vapor pressure of metastable β at 300 K (a) 1.23×10^{-5} atm (b) 2.11×10^{-5} atm (c) 3×10^{-5} atm (d) 1.16×10^{-4} atm (e) 1 atm

63. Which statement is correct?

(a) entropy of α phase at 0K is greater than 0
(b) entropy of β phase at 0 K is greater than 0
(c) Gibbs free energy of α phase is higher than that of β phase at 500 K
(d) Gibbs free energy of α phase is higher than that of β phase at 400 K
(e) slope of G (Gibbs free energy) vs T (temperature) is positive

Questions for (64-65)

It is desired to deposit a layer of SnO_2 (s) by chemical vapor deposition using $SnBr_4$ (g) and H_2O (g) in accord with the reaction: $SnBr_4 (g)+2H_2O(g)=SnO_2 (s)+ 4HBr (g)$

The standard Gibbs free energy of this reaction at 1000 K is 4600 cal/mole.

64. What is the initial driving force for the reaction (ΔG) for an input gas composition of 75 % Ar carrier gas, 10 % H_2O, 10 % $SnBr_4$, and 5 % HBr (molar percent), with a total pressure of 1 atm at 1000K? (ln 625=6.43, ln 0.00001=-11.51

(a)-5490 (b)-3210 (c) 6673 (d) 2530 (e) 1285 (cal/mole)

65. What is the equilibrium constant for this reaction at 1000 K?

(a) exp (-6.375) (b) exp (3.187) (c) exp (-4.673) (d) 0 (e) exp (-2.315)

Questions for (66-67)

CaO and MgO form a simple eutectic system with limited ranges of solid solubility. The eutectic temperature is 2370 °C. Assuming that the solutes in the two solid solutions obey Henry's law with γ_{MgO} in CaO=6.23 and the solubility of CaO in MgO is 0.066 at 2300 °C.

66. What is the solubility of MgO in CaO at 2300 °C.

(a) 0.25 (b) 0.15 (c) 0.45 (d) 0.65 (e) 0.75

67. What is the activity of CaO at X_{CaO}=0.5

(a) 0.85 (b)0.15 (c) 0.25 (d) 0.65 (e) 0.45

Question for (68)-(70)

An FCC phase (β) of an element A has been prepared by deposition on an FCC substrate. The stable form of A is BCC (α) at 298 K. Combustion of the β and α phases of A to form AO(s) yields ΔH values at 298 K of -52.7 and -51.7 Kcal/mole, respectively. The third law entropies are 12.8 and 12.0 cal/mole-K at 298 K and densities are 7.6 and 7.0 g/cc for the β and α phases, respectively. The atomic weight is 70. Assume the volume change is independent of pressure.

68. Assume ΔCp=0, what is the transition temperature between α and β at P=1 atm.

(a) 1000 K (b) 1150 K (c) 925 K (d) 1325 K (e) 1250 K

69. What is activity of the α forms at 298K and 1 atm?

(a) 1 (b)0.95 (c) 1.15 (c) 0.8 (d) 1.25

70. What is the equilibrium transition pressure at 300 K?

(a) 0.52 atm (b) 39235 atm (c) 0.03 atm (d) 132 atm (e) 25 atm

Question for (71)-(72)

Sn obeys Henry's law in dilute liquid solutions of Sn and Cd and the Henrian activity coefficient of Sn, γ^0_{Sn}, varies with temperature as $\ln \gamma^0_{Sn} = -840/T + 1.58$

1 mole of liquid Sn and 99 moles of liquid Cd are mixed in an adiabatic enclosure, and the molar constant pressure heat capacity of the alloy formed is 30 J/K.

71. What is the mixing enthalpy?

(a) 100 J (b) 21.7 J (c) -69.8 J (d) -37.6 J (e) -137.6 J

72. What is the change in temperature?

(a) 5.22 K (b) 7.68K (c) 12.51 K (d) 2.33 K (e) 0.37 K.

Questions for (73)-(75)

Melts in the system Pb-Sn exhibit regular solution behavior. At 746 K activity of Pb is 0.055 in a liquid solution of X_{Pb}=0.1. (ln 55~ 4, ln10~ 2.3, ln 0.1~-2.3, ln 0.9~-0.11)

73. What is the activity coefficient of Pb?

(a) 0.055 (b) 0.0055 (c) 0.1 (d) 0.01 (e) 0.55

74. What is the enthalpy of mixing of the solution, calculated by using the quasi-chemical model?

(a) –316 J (b) -206 J (c) 735 J (d) -412 J (e) 749 J.

75. What is the entropy of mixing of the solution?

(a) 3.65 (b) 2.73 (c) 6.82 (d)9.52 (e) 12.63 J/K

76. What is the temperature at which pure Ag_2O decomposes to Ag metal and O_2 gas when heated in pure oxygen at 1 atm pressure? The temperature dependence of standard Gibbs formation free energy of Ag_2O is $\Delta G^o = -30500 + 60\,T$ joules

(a) 508 K (b) 368 K (c) 592 K (d) 1240 K (e) 293 K

77. Which statement is **incorrect**?

(a) The activity of a component in any state at T is defined as being the ratio of the fugacity of the substance in that state to its fugacity in its standard state.

(b) Raoult's law states that the vapor pressure exerted by a component i in a solution is equal to the product of the mole fraction of i in the solution and the vapor pressure of pure i at the temperature of the solution.

(c) In an A-B binary system, if the magnitude of the A-B attraction is less than those of the A-A and B-B attractions, the Henry's law line lies above the Raoult's law.

(d) If a component i obeys Henry's law, its activity is proportional to its molar fraction.

(e) If a component i obeys Raoult's law, its activity is equal to its molar fraction.

78. Which statement is **incorrect**?

(a) If two phases α and β co-exist in a solution, the chemical potential of component A is equal in both phases

(b) Complete solid solubility of components A and B typically requires that A and B have the same crystal structure and be of comparable atomic size, electronegativity and valency.

(c) The volume of an ideal solution is equal to the sum of the volumes of pure components.

(d) The heat of formation of an ideal solution is negative.

(e) The entropy of formation of an ideal binary solution is independent of the temperature of the solution.

79. The standard Gibbs formation free energy of $Sn_{(l)}$, $SnO_{2\,(s)}$ $SnO_{(s)}$ at 1000 K are 0, -89200, and -43900 cal/mole. The related chemical reaction is: $Sn_{(l)} + SnO_{2\,(s)} = 2SnO_{(s)}$ Eq (A)

Which statement is **<u>incorrect</u>**?

(a) These three substances cannot co-exist at 1000 K

(b) If three substances co-exist, the standard Gibbs free energy of Eq (A) is larger than zero.

(c) If three substances co-exist, the equilibrium constant is equal to 1.

(d) These three substances can co-exist at a specific temperature.

(e) The activity of pure Sn is equal to 1.

80. Which statement is incorrect for the quasi-chemical model of solutions?

(a) The model is applied to solutions of components which are considered to have equal molar volumes in the pure state

(b) The model is applied to solutions of components which have zero volume change on mixing.

(c) The interatomic forces are significant over a long distance.

(d) The energy of the solution is calculated by summing the atom-atom bond energy.

(e) This model describes that a sufficient condition of an ideal solution is that E_{AB} be the average of E_{AA} and E_{BB} in an A-B binary system.