All the vectors are **underlined** and in **bold**.

Make any assumption if you need in order to solve the following problems.

1. Suppose we have a hollow wave-guide of rectangular shape, with height a and width 3a, and we are interested in the propagation of TE waves.

(a) Calculate the cut-off frequencies of T_{10} mode. (5 points)

(b) Find out the frequency range of a traveling wave propagating with single mode excitation. (5 points)

2. Draw qualitatively the corresponding electric **E** and magnetic **B** fields for each condition described below.

(a) A stationary electric charge q at the origin [0,0,0] in Cartesian coordinates. (5 points)

(b) An electric charge q moving with constant velocity **V** along the positive **z** direction. (10 points)

3. Suppose that a very long solenoid with radius R, N turns per unit length, and constant current I, has a magnetic field **B** pointing **z** direction. Coaxial with the solenoid are two long cylindrical shells of length L. One inside the solenoid at radius a , carries a charge +Q uniformly distributed over its surface; the other, outside the solenoid at radius b, carries charge -Q. L is supposed to be much larger than b. The whole setup above is located inside an ultra-high vacuum chamber made with high permeability μ metal. The high μ metal is used to keep the magnetic field of the earth away from the chamber.

(a) Find out the divergence of the **D** field at radius r = (a + b)/2. (5 points)

(b) Find out the curl of the **E** field at radius r = (a + b)/2. (5 points)

(c) Find out the curl of the **B** field at radius r = a/2. (5 points)

(d) Find out the divergence of the **H** field at radius r = R. (5 points)

(e) Find out Maxwell stress tensor T_s on the cylindrical surface at radius b. (5 points)

4. An electron is placed in vacuum at a distance x from an infinite, grounded metal plate. (15 points)

(a) What is the induced charge on the metal? What is the force on the electron due to the coulomb attraction with the induced charge? What is the potential energy of the electron (relative to the infinity)?

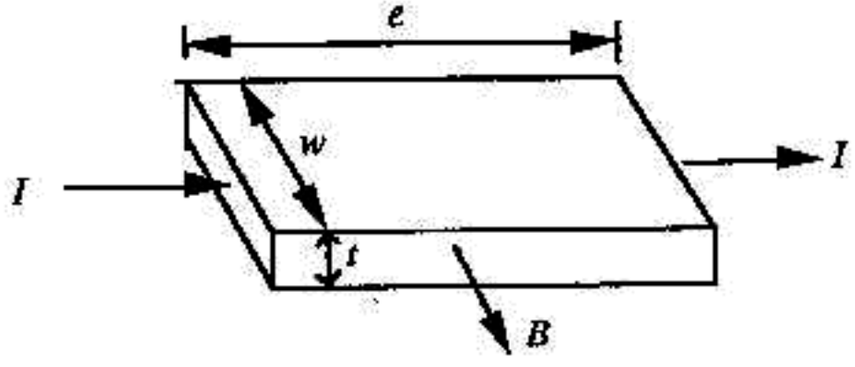
(b) If an electric field E (in $\bar{x}$ direction) is applied to the vacuum, the potential energy of the electron is modified. Please find the resulting potential energy of the electron and show that it has an extreme value of $-e\sqrt{\frac{eE}{4\pi\varepsilon_0}}$.

5. Find and show graphically the electric field everywhere produced by a uniformly polarized sphere of radius R. (Hint: you may choose the z axis to coincide with the direction of polarization $\vec{P}$.) (10 points)

6. A current I flows to the right through a rectangular metal bar, in the presence of a uniform magnetic field B pointing out of the page. (15 points)

(a) If the moving charges are positive, in which direction are they deflected by the magnetic field? Find the resulting potential difference between the top and bottom of the bar, in terms of B, v (the speed of the charges), and the relevant dimensions of the bar.

(b) How would your analysis change if the moving charges were negative?



7. Find the magnetic field a distance z above a long straight wire carrying a steady current I, using the Biot-Savart law. Check your answer by using the Ampere's law. (10 points)