國 立 清 華 大 學 命 題 紙

八十六學年度材料科學工程研究所(系)(所) 第二 組碩士班研究生入學考試

科目 電 磁 學 科號 2003 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Suppose that a linear dielectric sphere of radius R is uniformly polarized. The polarization $\underline{P}$ is along the $\underline{z}$ direction ($\underline{P}$ and $\underline{z}$ are vector quantities.)
   (a) Find the volume bound charge density (3 points)
   (b) Find the surface bound charge density. (3 points)
   (c) Find the electric field outside the sphere. (3 points)
   (d) Find the divergence of electric field at the center of the sphere. (3 points)
   (e) Find the curl of electric field outside the sphere. (3 points)

2. Suppose that a copper sphere of radius R carries a charge Q.
   (a) Calculate the electric field outside the sphere (5 points)
   (b) Find the scalar potential inside and outside the sphere (5 points)
   (c) Find the total energy stored in the system. (5 points)

3. Suppose that an infinitely long Fe-Co-Ni rod of radius R is given a uniform longitudinal magnetization $\underline{M}$, i. e. the magnetization along the rod axis ($\underline{M}$ is a vector quantity.)
   (a) Find the volume bound current density. (3 points)
   (b) Find the surface bound current density. (3 points)
   (c) Find the magnetic field inside the rod. (3 points)
   (d) Find the curl of magnetic field outside the rod. (3 points)
   (e) Find the vector potential outside the rod. (5 points)
   (f) Find the Laplacian of the vector potential along the rod axis. (3 points)

八十六學年度材料科學工程研究所(產系(所) 乙二 組碩士班研究生入學考試

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4. If a magnetic induction field B is expressed by cylindrical coordinate (r, θ, z), two of its components are:

$$B_z = B_o r^2 t \quad \cos(2\pi z/L),$$

$$B_\theta = 0$$

Moreover, both the electric field E and magnetic induction field B are not functions of θ. Find B_r and E_θ. (15%)

5. If the time-varying factor is $e^{j\omega t}$, prove the differential equations of $\vec{H}$ and $\vec{J}$ in conductors to be

$$\nabla^2 \vec{H} = j\omega\mu\sigma \vec{H},$$

$$\nabla^2 \vec{J} = j\omega\mu\sigma \vec{J},$$

where σ and μ are the conductivity and permeability of the conductor respectively. (15%)

6. A long coaxial cable carries current I. The current I flows down the surface of the inner cylinder of radius a and back along the outer cylinder of radius b as shown in the figure. Find the energy stored in a section of length l. (20%)