八十五學年度材料科學工程研究所(系)(所) 甲二乙二 組碩士班研究生入學考試

科目 電磁學 科號 1903 2003 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. (10 points) (a) Prove that $\nabla^2 \frac{1}{|\vec{r}|} = -4\pi\delta(\vec{r})$.

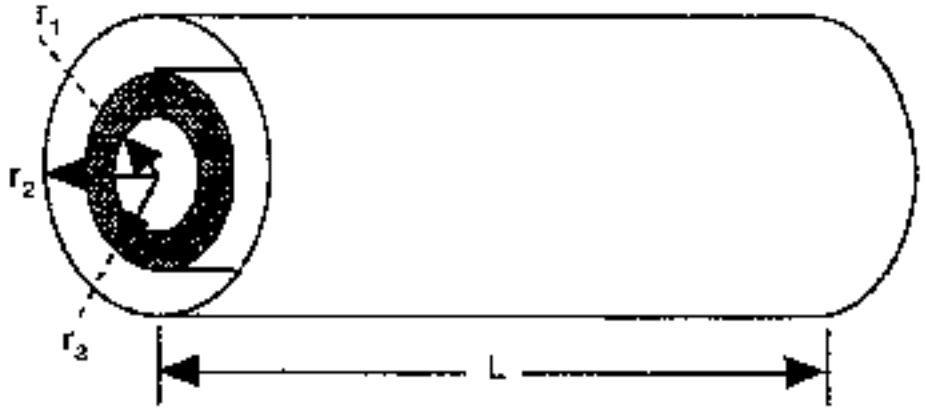
(b)What charge distribution gives the spherically symmetric potential $V(r) = e^{-\lambda r}/r$?

2. (15 points) A capacitor is made of two concentric cylinders of radius r_1 and r_2 ($r_1 < r_2$) and length $L >> r_2$. The region between r_1 and $r_3 = \sqrt{r_1 r_2}$ is filled with a circular cylinder of length L and dielectric constant K (the remaining volume is an air gap).

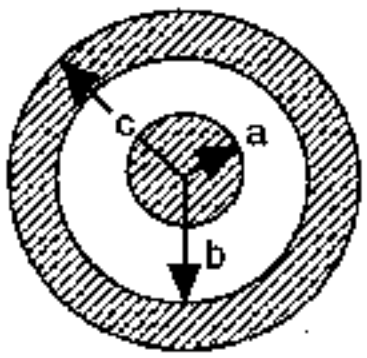
(a) What is the capacitance?

(b) What are the values of E, P, and D at a radius r in the dielectric ($r_1 < r < r_3$)? In the air gap ($r_3 < r < r_2$)? Assume a potential difference V between r_1 and r_2.

(c) How much mechanic work must be done to remove the dielectric cylinder while maintaining this constant potential difference between r_1 and r_2.



3. (10 points) A long coaxial cable (shown in cross section) has a uniform current I flowing in the center conductor into the paper, and the same current I flowing in the outer cylinder out of paper. Find the magnetic field $\vec{B}$ in each of the four regions: (i) $r < a$ (ii) $a < r < b$ (iii) $b < r < c$ (iv) $r > c$.



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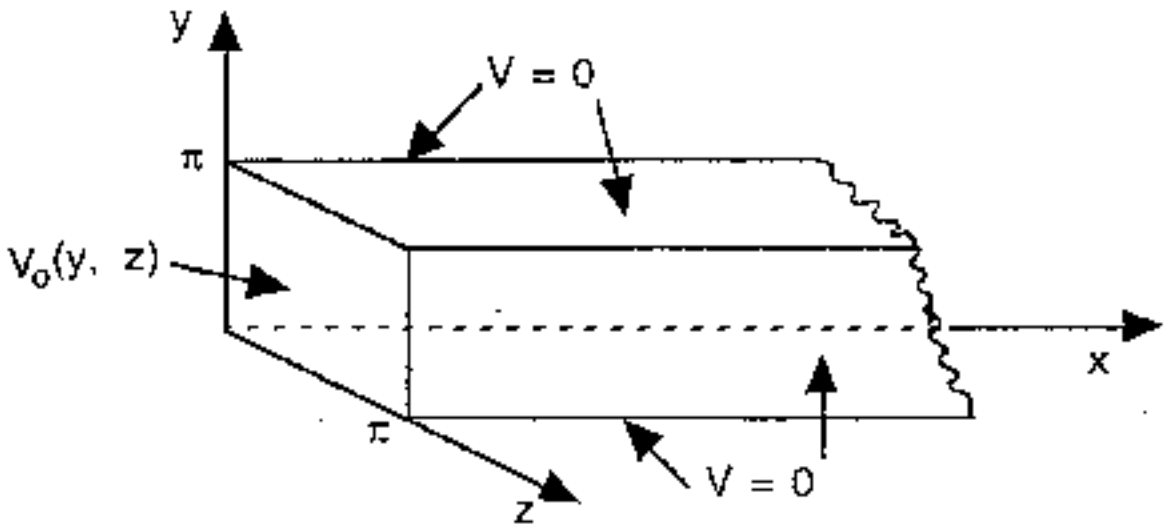
2003

4. (15 points) An infinitely long square metal pipe (sides π) is grounded, but one end, at x = 0, is maintained at a specified potential $V_0(y, z)$. Find the potential inside the pipe.



5. Derive the mathematical statement of local charge conservation, i. e. continuity equation for moving charges, from Ampere's law with Maxwell's correction. (10 points)

6. A battery, which supplies a constant emf $\mathcal{E}_0$, is connected to a circuit of resistance R and inductance L (i. e. LR circuit). If the resistance R becomes R/2 in the circuit, to what value the inductance should be changed in order to keep the "time constant" of the circuit unchanged? (10 points)

7. Imagine an infinitely long solenoid with radius R, N turns per unit length, and current I. (a) What is the energy stored in the solenoid per unit length? (5 points) (b) What is the momentum density stored in the solenoid? (5 points)

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8. Suppose that a monochromatic plane wave propagates in a conductor. Please explain the reason why the electromagnetic energy is not equally shared between electric and magnetic field. (10 points)

9. Suppose that we have an electric dipole with dipole moment $\mathbf{p} = p_0(2\cos\omega t\,\mathbf{i} + 3\sin\omega t\,\mathbf{j})$. Please calculate the total average power radiated through a sphere of radius r far away from the position of the dipole. (10 points)