科目：應用數學(2001)　校系所組：中央大學光電科學與工程學系照明與顯示科技碩士班
交通大學電子物理學系（丙組）
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清華大學物理學系
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[1] Consider a 4 dimensional metric linear space which basis vectors are given by $\{\mathbf{e}_0,\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\}=\{1,x,x^2,x^3\}$, with the inner product defined by $(u,v)=\int_{-1}^{1}u(x)v(x)dx$. For example, $(\mathbf{e}_1,\mathbf{e}_3)=\int_{-1}^{1}x^1x^3dx=\int_{-1}^{1}x^4dx=\frac{2}{5}$. Starting from $\mathbf{e}_0$, obtain the orthonormal basis functions using the Gram-Schmidt method. (10%)

[2] Cylindrical coordinates, $(r,\ \phi,\ z)$ are defined by $x=r\cos\phi$, $y=r\sin\phi$, and $z=z$. Obtain the Laplace operator $\nabla^2\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ in cylindrical coordinates. (10%)

參考用

[3] Obtain a second order homogeneous linear differential equation which two independent solutions are given by $f(x)$ and $g(x)$. (10%)

[4] Solve the following differential equations (20%)

(a) $\frac{d^2}{dx^2}y(x)+y(x)=\sin ax$,

(b) $\frac{d^2}{dx^2}y(x)-4\frac{d}{dx}y(x)+3y(x)=2e^{3x}$.

注意：背面有試題

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[5] Calculate $\int_{-\infty}^{+\infty} \frac{x^2+x+2}{x^4+10x^2+9} dx = ?$ (10%)

[6] Compute g(t)= $\mathrm{Re}\left\{\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{e^{i\omega t}}{\omega-\omega_0-i\nu}d\omega\right\}$ for both positive and negative t, where "Re" denotes the real part, ω_0 and ν are positive constants. Sketch your results assuming $\omega_0 \gg \nu$. (10%)

[7] Consider the rectangular region of $0 \le x \le 2,\ 0 \le y \le 3$. Find the eigenvalues and eigenfunctions that satisfy $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial x}+\lambda u(x,y)=0$ with $u(x,y)=0$ on the boundary. (Hint : let $u(x,y)=f(x)\sin(\frac{n\pi y}{3})$) (20%)

[8] Expand the Dirac function $\delta(x-t)$ in Fourier series. (10%)