

科目：應用數學(2001)

校系所組：中央大學光電科學與工程學系照明與顯示科技碩士班

交通大學電子物理學系（丙組）

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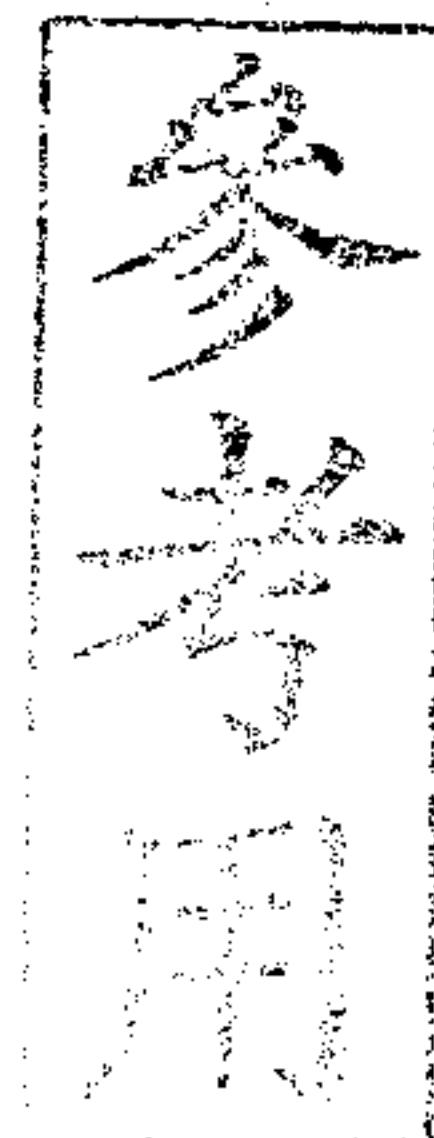
- [1] Consider a 4 dimensional metric linear space which basis vectors are given by $\{\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\} = \{1, x, x^2, x^3\}$, with the inner product defined by

$(u, v) = \int_{-1}^1 u(x)v(x)dx$. For example, $(\mathbf{e}_1, \mathbf{e}_3) = \int_{-1}^1 x^1 x^3 dx = \int_{-1}^1 x^4 dx = \frac{2}{5}$. Starting

from \mathbf{e}_0 , obtain the orthonormal basis functions using the Gram-Schmidt method. (10%)

- [2] Cylindrical coordinates, (r, ϕ, z) are defined by $x = r \cos \phi$,

$y = r \sin \phi$, and $z = z$. Obtain the Laplace operator $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in cylindrical coordinates. (10%)



- [3] Obtain a second order homogeneous linear differential equation which two independent solutions are given by $f(x)$ and $g(x)$. (10%)

- [4] Solve the following differential equations (20%)

(a) $\frac{d^2}{dx^2} y(x) + y(x) = \sin ax$,

(b) $\frac{d^2}{dx^2} y(x) - 4 \frac{d}{dx} y(x) + 3y(x) = 2e^{3x}$.

注意：背面有試題

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[5] Calculate $\int_{-\infty}^{+\infty} \frac{x^2 + x + 2}{x^4 + 10x^2 + 9} dx = ?$ (10%)

[6] Compute $g(t) = \operatorname{Re} \left\{ \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{e^{i\omega t}}{\omega - \omega_0 - iv} d\omega \right\}$ for both positive and

negative t , where "Re" denotes the real part, ω_0 and v are positive constants. Sketch your results assuming $\omega_0 \gg v$. (10%)

[7] Consider the rectangular region of $0 \leq x \leq 2$, $0 \leq y \leq 3$. Find the eigenvalues and eigenfunctions that satisfy $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} + \lambda u(x, y) = 0$ with $u(x, y) = 0$ on the boundary. (Hint : let $u(x, y) = f(x) \sin(\frac{n\pi y}{3})$) (20%)

[8] Expand the Dirac function $\delta(x-t)$ in Fourier series. (10%)

