國　立　清　華　大　學　命　題　紙

98 學年度 奈米工程與微系統研究所 系（所） 組碩士班入學考試

科目 工程數學 科目代碼 1803 共 2 頁第 1 頁 *請在【答案卷卡】內作答

1. (1) Find the motion of the mass-spring system in the Fig.1 with mass 0.125 kg, damping 0, spring constant 1.125 kg/sec^2 and driving force cos t– 4 sin t N, assuming zero initial displacement and velocity. For what frequency of the driving force would you get resonance?

   (10 分)

   Remark: $my'' + cy' + ky = r(t)$

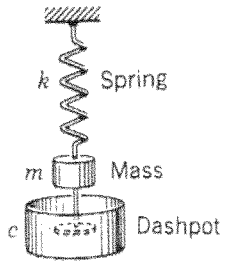

Fig. 1. Mass–spring system

   (2) Find the steady-state solution of the system in Fig.1 when m = 1, c = 2, k = 6 and the driving force is sin 2t + 2 cos 2t.

   (10 分)

   (3) In Fig.1, let m = 1, c = 4, k = 24 and $r(t)$ = 10 cos ωt. Determine ω such that you get the steady-state vibration of maximum possible amplitude. Determine this amplitude. Then find the general solution with this ω.

   (10 分)

   (4) In Problem. 1-(2), find the solution corresponding to initial displacement =1 and initial velocity = 0.

   (10 分)

2. Using the Laplace transformation, find *y(t)* satisfying the given equation and conditions.

(1) $y'' + 9y = 1.8u(t-3), \quad y(0) = 0,\ y'(0) = 0$ (6 分) (remark: u(t) is step function)

(2) $y'' + 4y' + 4y = 4\cos t + 3\sin t, \quad y(0) = 1,\ y'(0) = 0$ (6 分)

(3) $y'' + 4y = \delta(t-\pi) - \delta(t-2\pi), \quad y(0) = 0,\ y'(0) = 2$ (6 分) (remark: δ(t) is unit impulse function)

(4) $y'' - 4y' + 4y = (3t^2 + 2)e^t, \quad y(0) = 20,\ y'(0) = 34$ (6 分)

(5) $y(t) = \cosh 3t - 3e^{3t}\int_0^t y(\tau)e^{-3\tau}d\tau$ (6 分)

3. Find solutions *u(x,y)* of the following equations using separating variables.

(1) $u_x + u_y = 0$ (6 分)

(2) $u_x = u_y$ (6 分)

(3) $u_x + u_y = 2(x+y)u$ (6 分)

(4) $xu_x = yu_y$ (6 分)

(5) $u_{xx} + u_{yy} = 0$ (6 分)