1. (a) Solve the first-order differential equations

   $$x^2(y+1)dx + y^2(x-1)dy = 0$$ (10%)

   (b) Solve the differential equations where λ is real.

   $$y''(x) + \lambda y = 0, \ y(0) = y'(\pi) = 0, \ 0 \le x \le \pi$$ (10%)

   (c) Solve the simultaneous differential equations
   $$\dot{y}_1 - 3y_1 = y_2$$
   $$\dot{y}_2 - y_2 = -y_1$$ (15%)

2. Give the periodic function
   $$f(t) = \begin{cases} \sin t & 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases}$$
   Find the Laplace transform of $[f(t)]$. (10%)

3. (a) Let $p(s)$ is a polyminal in s. Show that if λ is an eigenvalue of a square matrix A with eigenvector x,

   then $p(\lambda)$ is an eigenvalue of $p(A)$ with the same eigenvector x. (10%)

   (b) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Compute A^k for any integer k. (15%)

4. Evaluate $\iint_S \mathbf{F} \cdot \mathbf{n} dS$, where $\mathbf{F} = z\mathbf{i} + x\mathbf{j} - 3y^2 z\mathbf{k}$, S is the surface of the cylinder $x^2 + y^2 = 16$ included in the

   first octant between z = 0 and z = 5, and n is the unit vector of the surface S. (15%)

5. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$, where a is constant. (7%)

   (b) Use the result of (a) to evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{\sin \alpha a \cos \alpha x}{\alpha} d\alpha$. (8%)