1. Modeling and Equations of a Mechano-Receptor Cell

Deep under the skin of your hands and feet are nerves called "mechano-receptors" that help sense touching. Figure 1 depicts a cross section of a human Pacinian corpuscle mechano-receptor. Depressions of the surface of the skin are transmitted to the receptor through layers of fluid and tissue. The signal $y(t)$ sent to the nervous system is proportional to the deformation of the receptor.

After your graduation, you work at a Bio-company. Your colleague, David, modeled it as the system in Fig 2. Your boss asks you to take care of this job to analyze above system because David got fired. You are working alone on the following analysis.

(a) The schematic in Fig 2 can be simplified by replacing the three springs by one spring with spring constant k_e, and replacing two dampers by one damper with damping constant b_e. Determine k_e and b_e in terms of k_i and b_i. (5%)

(b) Determine $F^{p/damper}$, the vector force on P from the equivalent damper, b_e, in terms of y, dy/dt, d^2y/dt^2, y_{input}, dy_{input}/dt, d^2y_{input}/dt^2, b_e. (5%)

(c) Consider the simplified model and assume $y=0$ at equilibrium. Write the equation governing the motion of P in terms of y, dy/dt, d^2y/dt^2, A, Ω, t, m, k_e, b_e. (5%)

(d) By running laboratory experiments, you got $k_e=100$ N/m and Natural frequency $\omega_n = 2500$ rad/sec. Find the equivalent mass m. (5%)
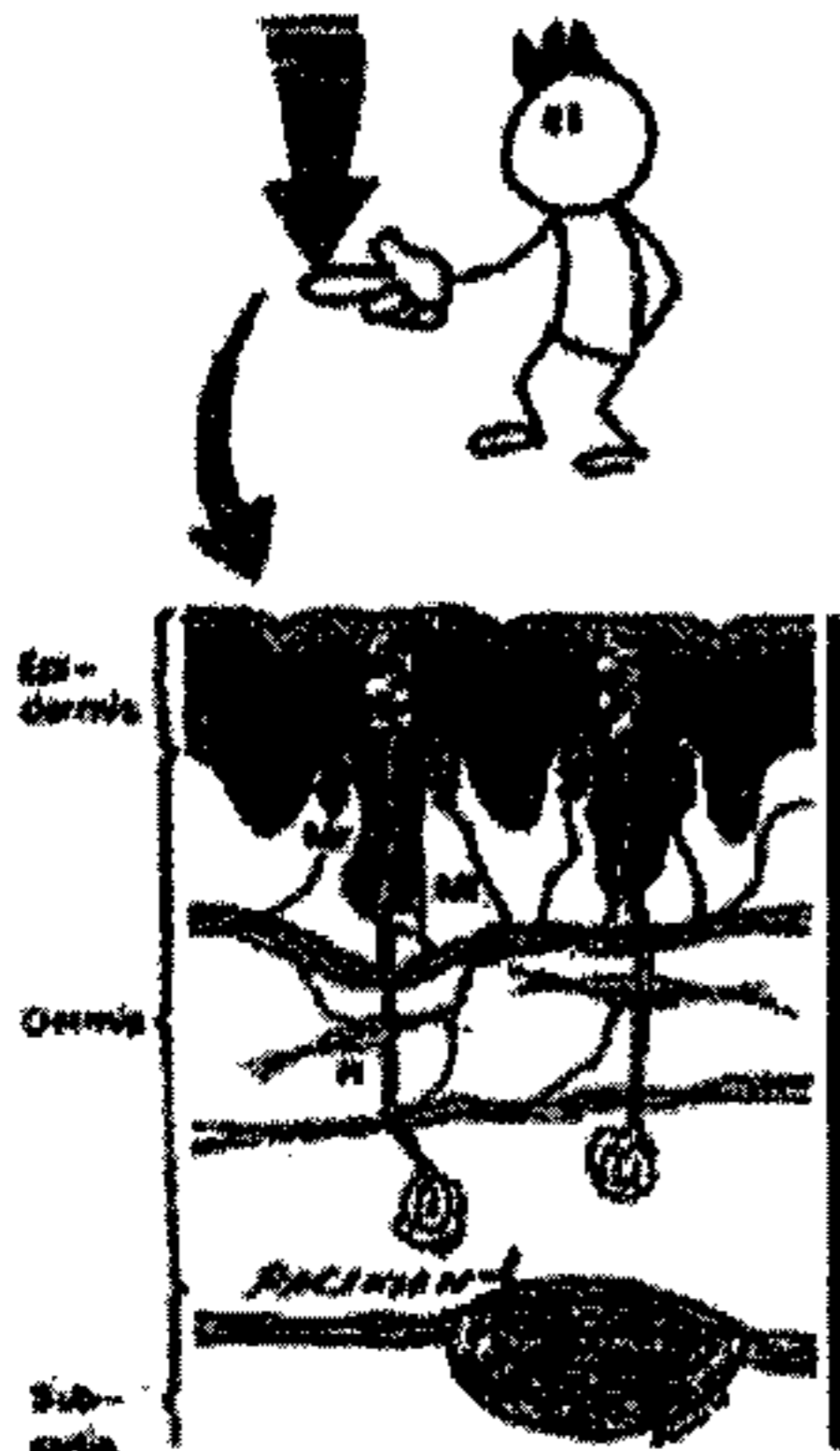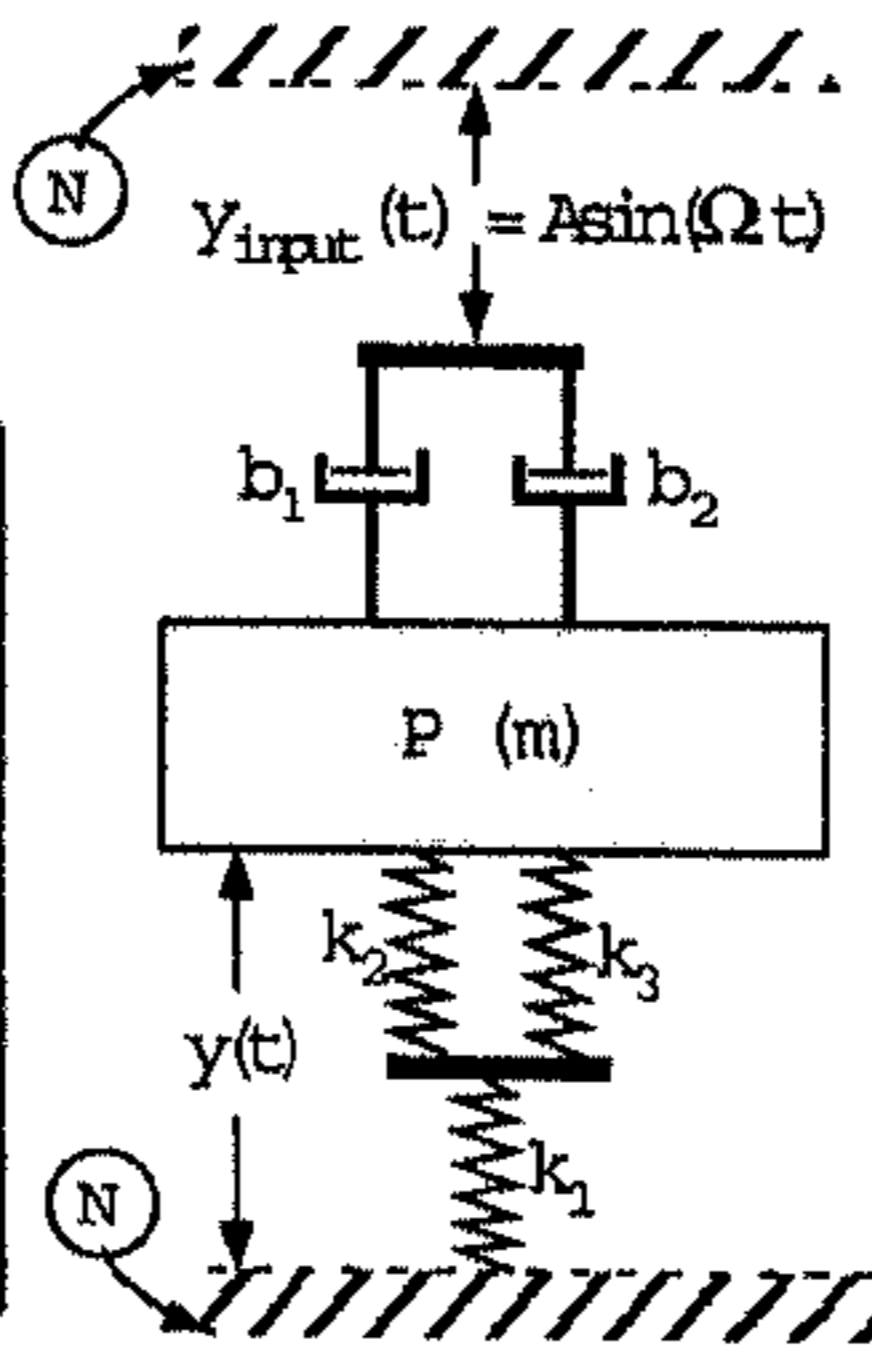

Figure 1   Figure 2

2. (a) A microscope has a 30X eyepiece. The objective and eyepiece are separated by 20 cm. You discover that you have to place the object that you wish to observe 1 cm from the objective lens. What is the total magnification for this microscope?(5%)

(b) You want to use a right angle prism to reflect light from the horizontal to the vertical direction as shown. Air is surrounded around this prism. (Air has n=1) What materials would you use from the following choices? (water has n=1.3, glass n=1.5, diamond n=2.2.) (5%)
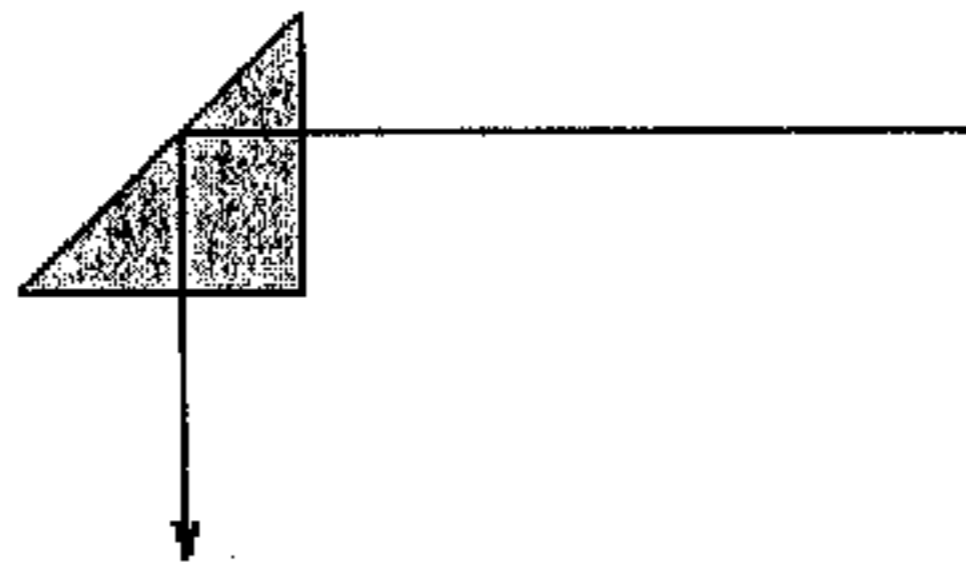
3. A spherical balloon of volume 1000cm^3 contains helium at an (inside) pressure of 1.4×10^5 Pa. How many moles of helium are in the balloon, if each helium atom ahs an average kinetic energy of 3.6×10^{-22}J? (15%)

4. A Ping-Pong ball has a diameter of 3.5 cm and average density of 0.09 g/cm^3. What force would be required to hold it completely submerged under water? (15 %)
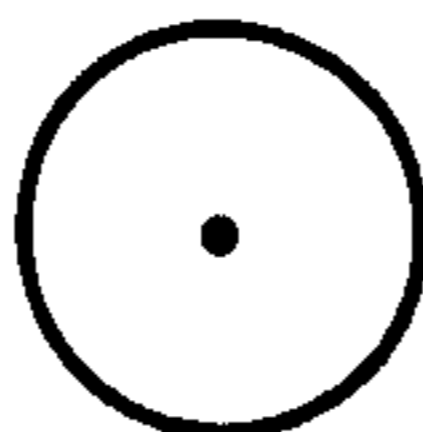
5. [Electrostatic Force]
   A. Please write down the governing equation of Coulomb's Law. (2%)
   B. Four charges with charge quantities of Q_1, Q_1, Q_1, $-Q_2$, respectively, exist in the free space (e.g. air). Find out an arrangement so that all four charges won't stand still. Also prove your idea. (4%)

6. [GAUSS Law]
   Draw a schematic of charge distribution and electric field distribution when a charge is placed right in the center of 2D conductive O-ring. (8 %)
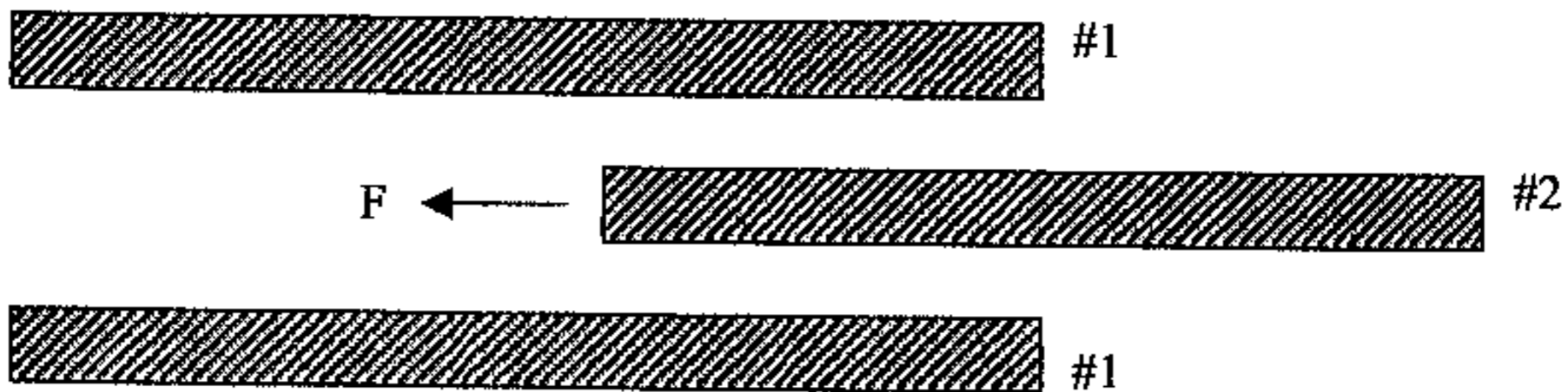
7. [Capacitance]

Please calculate output force of an electrostatic comb-drive (shown below). The comb-drive consists of three perfect conductors where #1 conductors are fixed and #2 conductor is movable. Hint that force is derivative of stored (electrostatic) energy with respect to displacement. (8 %)

8. [Circuitry]

Derive the time constant of discharging a capacitor. The following circuit is composed of one battery E, one switch S, one resistor **R**, and one capacitor **C**. (8 %)
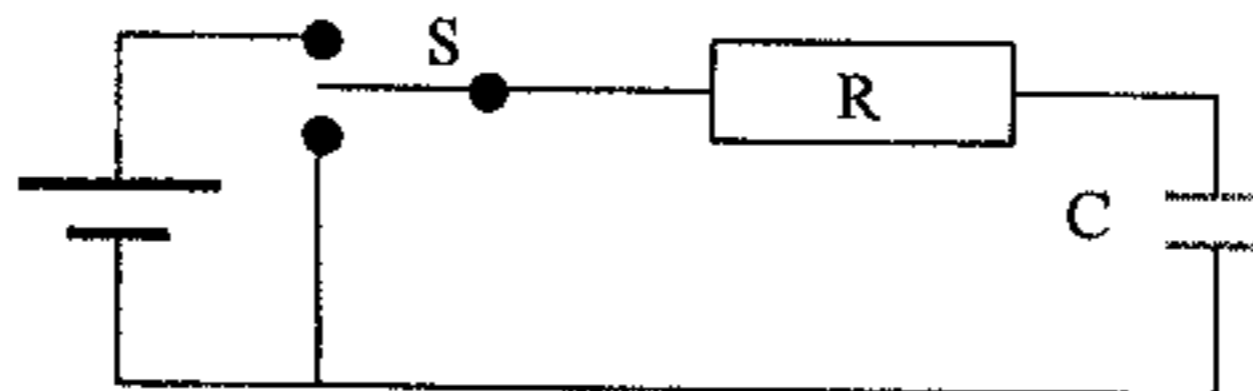
9. [Inductive Electric Field]

The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field **B** (directed out of the page) at the same constant velocity. As the four loops move through the field, please rank them according to the maximum magnitude of the emf and the induced current, respectively. (10 %)