國立清華大學命題紙

九十二學年度 微機電系統工程研究 (系)所 組碩士班研究生招生考試

科目 應用數學 科號 2202 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Solve the following first-order differential equation:

$$y'(x) = \frac{2y}{x(y-1)}$$ (10%)

2. Solve the following second-order differential equation:

$$(x+1)^2 y''(x) + 2(x+1)y'(x) - 4y(x) = 2x+1$$ (15%)

3. Find the eigenvalues and the corresponding eigenvectors for the matrix A.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$ (15%)

4. (a) Prove $\mathcal{L}[\dot{f}(t)] = sF(s) - f(0)$ if the Laplace transform of $f(t)$ is $\mathcal{L}[f(t)] = F(s)$. (5%)

   (b) Find the inverse Laplace transform, $\mathcal{L}^{-1}\left[\frac{3s+1}{(s-1)(s^2+1)}\right]$ (5%)

5. If $\overline{V}$ is a vector function, show the following

   (1) $\nabla \bullet (\nabla \times \overline{V}) = 0$. (5%)

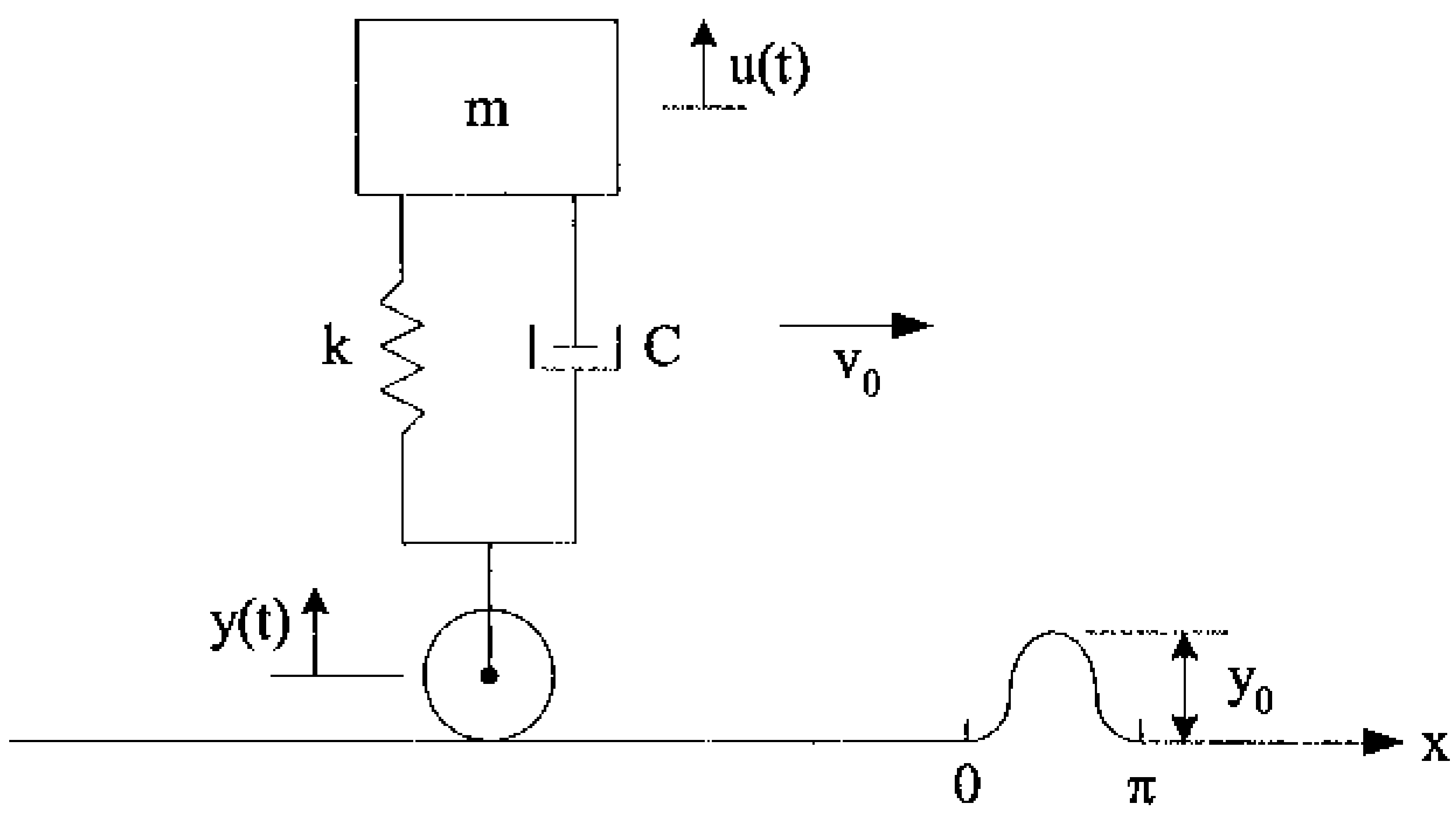
   (2) $(\overline{V} \bullet \nabla)\overline{V} = (\nabla \times \overline{V}) \times \overline{V} + \nabla(V^2/2)$. (5%)

   (3) $(\nabla \times \overline{V}) \times \overline{V}$ is normal to $\overline{V}$. (5%)

6. Evaluate the integrals $\int_{-\infty}^{\infty} \frac{\cos kx}{(x-a)^2+b^2}dx$ and $\int_{-\infty}^{\infty} \frac{\sin kx}{(x-a)^2+b^2}dx$ for k>0 by using Fourier Transform.

   (Hint: $e^{ikx} = \cos kx + i \sin kx$) (15%)

7. A monocycle shown below moves at a constant velocity v_0 hitting a bump along x direction. Assume the mass of the suspension and wheel assemble is negligible.



(a) Please derive the second order govern equation of this system as below

$$\frac{d^2u(t)}{dt^2}+\frac{d[u(t)]}{dt}+5u(t)=y(t) \qquad where \begin{cases} y=y_0\sin^2(8t) & (0<x<\pi/8) \\ \ \ =0 & (x<0, x>\pi/8) \end{cases}$$

- The relation between spring constant k and mass m is $k/m=5$, the damping constant C and mass m is $C/2m=1$, and the constant velocity is $v_0=8$.
- The bump condition: $\begin{cases} y=y_0\sin^2 x & (0<x<\pi) \\ \ \ =0 & (x<0, x>\pi) \end{cases}$.

(Hint: Start from relative parameter u(t)-y(t). Find relationship between x and t, then make derived PDE to be u(t) only equation.) (3%)

(b) Solve the PDE you derived above if $C=0$ (no damping case). (14%)

(c) Continue from (b), and find $u(t)/y_0$ if initial conditions are $u(0)=0$ and $\left.du(t)/dt\right|_{t=0}=0$. (3%)