國 立 清 華 大 學 命 題 紙

九十學年度 微機電系統工程研究 所__ 組碩士班研究生招生考試

科目 應用數學 科號 2201 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. Solve the differential equations

   (a) $(2x^2+y)dx+(x+2x^2y-x^4y^3)dy=0$ (10%)

   (b) $y''(x)+y(x)=\sin x+xe^x$ (15%)

2. (a) Let V be the vector space of 2 x 2 matrices over **R**. Determine whether the matrices **A**, **B**, **C** $\in$ V are dependent where:

   $$\mathbf{A}=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}.$$ (5%)

   (b) Let **A** be an n x n matrix. Show that for all n x 1 vector **x**, $\left|\mathbf{x}^T\mathbf{A}\mathbf{x}\right|\le\|\mathbf{A}\|\|\mathbf{x}\|^2$, where $\|\bullet\|$ denotes a norm. (5%)

3. The matrix **A** is a 2 x 2 constant matrix with a pair of complex conjugate eigenvelues $\alpha+j\beta$ and $\alpha-j\beta$. Find the transformation matrix **P** such that

   $$\mathbf{B}=\mathbf{P}^{-1}\mathbf{A}\mathbf{P}=\begin{bmatrix}\alpha & \beta\\ -\beta & \alpha\end{bmatrix}$$ (15%)

4. (a) Prove that the Fourier transform of the convolution product of f(t) and g(t) is given by

   $$\mathcal{F}[f(t)*g(t))]=\sqrt{2\pi}\mathcal{F}[f(t)]\mathcal{F}[g(t)]$$ (13%)

   (b) Determine the Fourier transform of the function

   $$f(t)=\frac{5e^{3it}}{t^2-4t+13}$$ (12%)

5. Write the solutions of the following boundary value problems

   $$\frac{\partial u}{\partial t}=3\frac{\partial^2 u}{\partial x^2}\quad(0<x<L,t>0)$$

   $$u(0,t)=u(L,t)=0\quad(t>0)$$

   $$u(x,0)=L[1-\cos(\frac{2\pi x}{L})]\quad(0<x<L)$$ (25%)