1. Prove that $f(x) = x^2$ is an increasing function on the interval $(0, \infty]$. (10 pts)

2. Pair (a) (b) (c) (d) with (e) (f) (g) (h). (5 pts)
   (a) $F(b) = x^b$
   (b) $g(a) = a^9$
   (c) $h(x) = \dfrac{2+x}{1-\sqrt{x}}$
   (d) $u(t) = 1 - t + 7t^4$
   (e) Exponential function
   (f) Power function
   (g) Algebraic function
   (h) Polynomial function

3. Evaluating $\lim_{x \to 0} \dfrac{\sqrt[3]{1-5x}+1}{x}$. (10 pts)

4. Find y' if $x^3 + y^3 = 6xy$. (10 pts)

5. (a) Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of
   $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. (10 pts)

   (b) In what directions does f change most rapidly at P_0, and what are the rates of change in these directions? (10 pts)

6. **Partial Derivative:** The plane x = 1 intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at $(1, 2, 5)$. (15 pts)
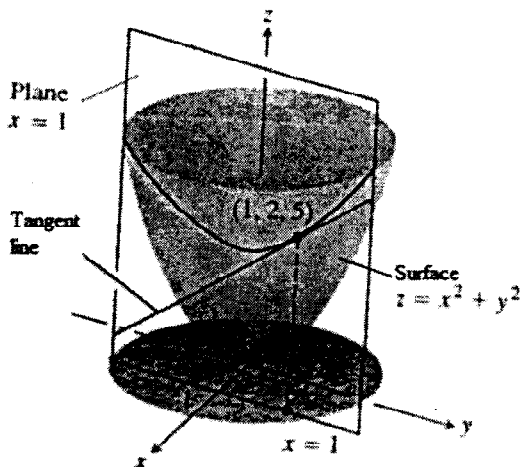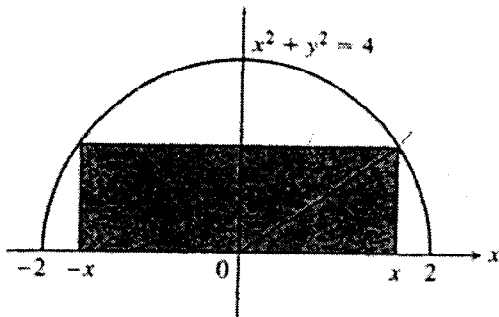
7. **Max-Min Problem**: A rectangle is to be inscribed in a semicircle of radius 2. What is the largest area the rectangle can have, and what are its dimensions? (15 pts)

8. Your metal fabrication company is bidding for a contract to make sheets of corrugated steel roofing. The cross sections of the corrugated sheets are to conform to the curve.

$$y = \left(\frac{x}{2}\right)^{\frac{2}{3}}, \ 0 \le x \le 2$$

If the roofing is to be stamped from flat sheets by a process that does not stretch the material, how wide should the original material be? (15 pts) The arc length of the curve y = f(x) from a to b is equal to $\int_{a}^{b}\sqrt{1+(\frac{dy}{dx})^2}\ dx$.