國立清華大學命題紙

99學年度 工業工程與工程管理學系工業工程組甲組 碩士班入學考試

科目 作業研究 科目代碼 1402 共 四 頁第 1 頁 *請在【答案卷卡】作答

注意事項: (1) 不得使用計算器。
(2) 請依題號順序作答。
(3) 答案必須寫在答案卷上, 並須依每一題規定的方式作答。
(4) 未依規定方式作答, 酌量扣分。

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1. True (T) or False (F)? 以下是非題每一小題2分, 但答錯一小題倒扣2分, 最多只扣到本題0分, 必須在答案卷畫出以下表格並在表格內填寫答案。

題目 1	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
答案										

(1) $(x_1, x_2) = (2,6)$ is an optimal solution of the following linear program

$$\begin{aligned} \text{Maximize} \quad & 3x_1 + 5x_2 \\ \text{subject to} \quad & x_1 \le 4 \\ & 2x_2 \le 12 \\ & 3x_1 + 2x_2 \le 18, \; x_1 \ge 0, x_2 \ge 0 \end{aligned}$$

and $(y_1, y_2, y_3) = (0, \frac{3}{2}, 1)$ is an optimal solution of its dual problem

$$\begin{aligned} \text{Minimize} \quad & 4y_1 + 12y_2 + 18y_3 \\ \text{subject to} \quad & y_1 + 3y_3 \ge 3 \\ & 2y_2 + 2y_3 \ge 5, \quad y_1 \ge 0, y_2 \ge 0, y_3 \ge 0. \end{aligned}$$

We can conclude that $(x_1, x_2, x_{new}) = (2, 6, 0)$ is an optimal solution of the following problem.

$$\begin{aligned} \text{Maximize} \quad & 3x_1 + 5x_2 + 5x_{new} \\ \text{subject to} \quad & x_1 + 3x_{new} \le 4 \\ & 2x_2 + 2x_{new} \le 12 \\ & 3x_1 + 2x_2 + x_{new} \le 18, \quad x_1 \ge 0, x_2 \ge 0, x_{new} \ge 0. \end{aligned}$$

(2) For any payoff table of a two-person zero-sum game, if mixed strategies are allowed, then there exists at least one stable solution.

(3) We can construct a linear program which has exact two distinct feasible

solutions.

(4) The feasible region of any given linear program must be a convex set.

(5) The optimal solution of a linear program may not be an extreme point.

(6) Once a primal LP has no feasible solution, we know that its dual is unbounded.

(7) The simplex method can solve the maximum flow problem.

(8) The following linear fractional program can be transformed into a linear program.

$$\text{Maximize} \quad f(\mathbf{x}) = \frac{\mathbf{cx} + c_0}{\mathbf{dx} + d_0}$$

$$\text{subject to} \quad \mathbf{Ax} \le \mathbf{b},\ \mathbf{x} \ge \mathbf{0}.$$

(9) In order to obtain a basic feasible solution, the surplus variables must be 0 in the two-phase method.

(10) In an LP model, adding an additional constraint can improve the value of the objective function.

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2. Consider the following linear program.

$$\begin{aligned} \text{Maximize} \quad & 5x_1 + 2x_2 + 3x_3 \\ \text{subject to} \quad & x_1 + 5x_2 + 2x_3 \le b_1 \\ & x_1 - 5x_2 - 6x_3 \le b_2,\ x_1, x_2, x_3 \ge 0. \end{aligned}$$

Let x_4, and x_5 be the slack variables of constraints 1 and 2 respectively. Specific constant values of b_1 and b_2 produce the following tableau.

Basic Variable	Coefficients of:						RHS
	z	x_1	x_2	x_3	x_4	x_5	
z	1	0	A1	A2	A3	0	D
x_1	0	1	B1	2	B2	0	30
x_5	0	0	C1	−8	C2	1	10

Determine b_1, b_2, A1, A2, A3, B1, B2, C1, C2, and D. 必須在答案卷畫出以下表格並在表格內填寫答案。

題目 2	b_1	b_2	A1	A2	A3	B1	B2	C1	C2	D
答案										

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3. Consider the following linear program.

$$\begin{aligned} \text{Maximize} \quad & c_1x_1 + c_2x_2 \\ \text{subject to} \quad & a_{11}x_1 + a_{12}x_2 \le b_1 \\ & a_{21}x_1 + a_{22}x_2 \le b_2 \\ & a_{31}x_1 + a_{32}x_2 \le b_3,\ x_1, x_2 \ge 0. \end{aligned}$$

Let x_3, x_4, and x_5 be the slack variables of constraints 1, 2, and 3 respectively. The optimal tableau is given as follows.

Basic Variable	Coefficients of:						RHS
	z	x_1	x_2	x_3	x_4	x_5	
z	1	9/2	0	0	0	5/2	40
x_3	0	1	0	1	0	0	4
x_2	0	3/2	1	0	0	1/2	9
x_4	0	−3	0	0	1	−10	6

Solve the following linear program with one additional constraint.

$$\begin{aligned} \text{Maximize} \quad & c_1x_1 + c_2x_2 \\ \text{subject to} \quad & a_{11}x_1 + a_{12}x_2 \le b_1 \\ & a_{21}x_1 + a_{22}x_2 \le b_2 \\ & a_{31}x_1 + a_{32}x_2 \le b_3 \\ & 2x_1 + 3x_2 \le 24,\ x_1, x_2 \ge 0. \end{aligned}$$

Determine the optimal value and the optimal primal solution.

必須在答案卷畫出以下表格並在表格內填寫答案。

題目 3	optimal value	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*
答案						

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4. Explain the following terminology in queueing system.:
   (1) *M/G/s/K* Model (8%)
   (2) Jackson Network (7%)

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5. Airplanes arrive for take-off at the runway of an airport according to a Poisson process at a mean rate of 20 per hour. The time required for an airplane to take off has an exponential distribution with a mean of 2 minutes, and this process must be completed before the next airplane can begin to take off. Because a brief thunderstorm has just begun, all airplanes which have not commenced take-off have just been grounded temporarily. However, airplanes continue to arrive at the runway during the thunderstorm to await its end. Assuming steady-state operation before the thunderstorm, determine the expected number of airplanes that will be waiting to take off at the end of the thunderstorm if it lasts 30 minutes.

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6. Customers arrive at a fast food restaurant with one server according to a Poisson process at a mean rate of 20 per hour. The server has just resigned, and the two candidates for the replacement are X (fast but expensive) and Y (slow but inexpensive). Both candidates would have an exponential distribution for service times with X having a mean of 1.2 minutes and Y having a mean of 1.5 minutes. Restaurant revenue per month is given by \$3,000/W where W is the expected waiting time (in minutes) of a customer in the system.
   (1) Determine the expected waiting time of a customer in the system by hiring either X or Y. (10%)
   (2) Determine the upper bound on the difference in their monthly compensations that would justify hiring X rather than Y. (10%)