國 立 清 華 大 學 命 題 紙

98 學年度工業工程與工程管理學系 甲 組碩士班入學考試

科目 作業研究 科目代碼 1402 共 2 頁第 1 頁 * 請在【答案卷卡】內作答

1. The birth and death processes are key processes in Queueing Theory, describe
   (1) (5%) The birth and death process for M/M/1/k.
   (2) (5%) The birth and death process for M/M/3.

2. Quick Lube, Inc. operates a fast lube and oil change garage. On a typical day, customers arrive at the rate of three per hour, and lube jobs are performed at an average rate of one every 15 minutes. The mechanics operate as a team on one car a time. Assume Poisson arrivals and exponential service, determine the:
   (1) (5%) Utilization of the lube team.
   (2) (5%) Average number of cars in line.
   (3) (5%) Average time a car waits before it is lubed.
   (4) (5%) Total time it takes a car to go through the system (i.e., Waiting in line plus lube time).

3. (20%) Express Vending Inc. supplies vended food to a large university. Because students kick the machines at every opportunity out of anger and frustration, management has a constant repair problem. The machines break down on an average of three per hour, and the breakdowns are distributed in a Poisson manner. Downtime cost the company $25/hour per machine, and each maintenance worker gets $4 per hour. One worker can service machines at an average rate of five per hour, distributed exponentially; two workers, working together, can service seven per hour, distributed exponentially; and a team of three workers can do eight per hour, distributed exponentially. What is the optimum maintenance crew size and total cost for servicing the machines? Analyze your decision in detail.

4. Assume a given basis of the vectors

$$P_1 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, P_3 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

(a). (15%) Determine an LU representation of the matrix associated with this basis and determine the linear combination of the basis vectors that equals vector $P_4 = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$

(b). (15%) Taking this example to explain and compare Gaussian Elimination Method and Gauss-Jordan Method by estimating their total numbers of arithmetic operations.

5. A manufacturer wishes to ship a number of units of an item from *m* warehouses to *n* retail stores. Each store requires a certain number of units of the item, while each warehouse can supply up to a certain amount. If the total amount available is equal to the total required,

(a). (5%) try to formulate a mathematical model to determine the amount of the item shipped from each warehouse to each store with the minimum total shipping cost

(b). (5%) write the dual program of this model.

(c). (10%) show that your model is always feasible.