國　立　清　華　大　學　命　題　紙

97 學年度工業工程與工程管理學系（所）甲組 碩士班入學考試

科目 作業研究 科目代碼 1402 共 4 頁第 1 頁 *請在【答案卷卡】內作答

注意事項：(1) 不得使用計算器。
(2) 請依題號順序作答。
(3) 答案必須寫在答案卷上，並須依每一題規定的方式作答。
(4) 未依規定作答，酌量扣分。

10%

1. True (T) or False (F)? 以下是非題每一小題 2 分，但答錯一小題倒扣 2 分，最多只扣到本題 0 分，必須在答案卷畫出以下表格並在表格內填寫答案。

題目 1	(1-1)	(1-2)	(1-3)	(1-4)	(1-5)
答案					

(1-1) The primal problem must always be of the maximization type.

(1-2) Changes in the right side of the coefficients of a linear program can affect only the right side of its optimal tableau, that is, feasibility.

(1-3) The following problem is equivalent to a linear program.

Minimize the maximum of $\{ | c_{11}x_1 + c_{12}x_2 - d_1 |, | c_{21}x_1 + c_{22}x_2 - d_2 | \}$

subject to $a_{11}x_1 + a_{12}x_2 \le e_1$

$a_{21}x_1 + a_{22}x_2 \le e_2, \quad x_1, x_2 \ge 0.$

(1-4) The number of basic variables in Phase I is the same as in Phase II.

(1-5) If the slack variable associated with a resource is positive, the unit worth of the resource may not equal zero.

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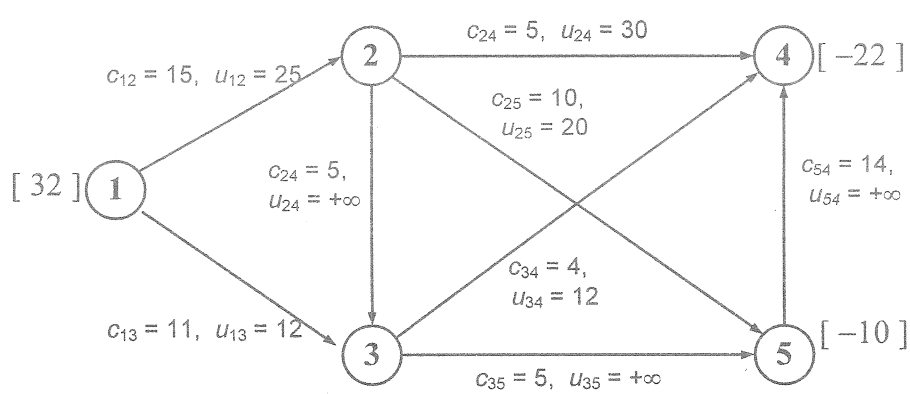
2. For the following minimum cost flow problem, let x_{ij} be the amount of flow from node i to node j. Find a basic feasible solution with basic variables: $x_{12}, x_{24}, x_{25}, x_{34}$.



必須在答案卷畫出以下表格並在表格內填寫答案。

題目 2	x_{12}	x_{13}	x_{23}	x_{24}	x_{25}	x_{34}	x_{35}	x_{54}
答案								

15%

3. Consider the following parametric linear programming problem with parametric change on the right-hand side.

$$\begin{aligned} \textit{Maximize} \quad z(t) = \; & 3x_1 + 2x_2 \quad + 5x_3 \\ \textit{subject to} \quad & x_1 + 2x_2 + x_3 \le 40 - t \\ & 3x_1 \quad\quad + 2x_3 \le 60 + 2t \\ & x_1 + 4x_2 \quad\quad \le 30 - 7t \\ & x_j \ge 0, \quad j = 1, 2, 3. \end{aligned}$$

Let x_4, x_5, and x_6 be the slack variables of constraints 1, 2, and 3 respectively. The optimal tableau for $t = 0$ is given as follows.

Basic Variable	Coefficients of:							RHS
	z	x_1	x_2	x_3	x_4	x_5	x_6	
z	1	w_1	0	0	w_2	w_3	0	w_4
x_2	0	−1/4	1	0	1/2	−1/4	0	5
x_3	0	3/2	0	1	0	1/2	0	30
x_6	0	2	0	0	−2	1	1	10

(3-1) Solve w_1, w_2, w_3, and w_4. Show your calculation. (5%)

必須在答案卷畫出以下表格並在表格內填寫答案。

題目(3-1)	w_1	w_2	w_3	w_4
答案				

(3-2) Solve the above parametric programming problem for $0 \le t < +\infty$. Show your calculation. (10%)
必須在答案卷畫出以下表格並在表格內填寫答案。

題目(3-2)	*range of t*	$x_1(t)$	$x_2(t)$	$x_3(t)$	optimal value $z(t)$
答案					

15%

4. Consider the following payoff table of a two person zero-sum game.

		Player 2		
		1	2	3
Player 1	1	c_{11}	c_{12}	c_{13}
	2	c_{21}	c_{22}	c_{23}

If mixed strategies are allowed, show that the pair of mixed strategies that is optimal according to the minimax criterion provides a stable solution with

expected payoff for player 1 = expected loss for player 2.

10%

5. Machines in a factory break down at an exponential rate of six per hour. There is a single repairman who fixes machines at an exponential rate of eight per hour. The cost incurred in lost production when machines are out of service is $100 per hour per machine. What is the average cost rate incurred due to failed machines?

20%

6. A supermarket has two exponential checkout counters, each operating at rate μ. Arrivals are Poisson at rate λ. The counters operate in the way that one queue feeds both counters. Let P_n denote the proportion of time there are n customers in the system.

(6-1) Draw a rate diagram for this queueing system. (5%)

(6-2) Develop the balance equations. (5%)

(6-3) Solve for P_n. (Express P_n explicitly in terms of λ, n, and μ only). (10%)

20%

7. Consider a system of two infinite queues in series, where each of the two service facilities has a single server. All service times are independent and have an exponential distribution, with a mean of 3 minutes at facility 1 and 4 minutes at facility 2. Facility 1 has a Poisson input process with a mean rate of 10 per hour.

(7-1) Find the steady-state distribution of the number of customers at facility 1 and then at facility 2. (5%)

(7-2) Show the product form solution for the joint distribution of the number at the respective facilities. (5%)

(7-3) What is the probability that both servers are idle? (5%)

(7-4) Find the expected total number of customers in the system and expected total waiting time (include service times) for a customer. (5%)