註：不得使用計算器；滿分 100 分。

1. (10 pts.) A survey was made of 100 customers in a bookstore. Sixty of the 100 indicted they visited the store because of a newspaper advertisement. The remainder had not seen the ad. A total of 40 customers made purchases; of these customers, 30 had seen the ad. What is the probability that a person who did not see the ad made a purchase? What is the probability that a person who saw the ad made a purse?

2. (15 pts.) In a certain factory, machines A, B, C, and D produce the same product. Of the total production, machine A, B, C, and D produce 10%, 20%, 30%, and 40%, respectively. The proportions of defective items produced by machines A, B, C, and D are 0.1%, 0.05%, 0.5% and 0.2%, respectively. An item selected at random is found to be defective. What is the item was produced A? by B? by D?

3. (15 pts.) Given that the probability distribution function of random variable X is

$$f_X(x) = \begin{cases} 0, & x < 0 \\ 1 - pe^{-x}, & x \geq 0, 0 < p < 1. \end{cases}$$

Find the moment generating function of X and use it to find the variance of X.

4. (60 pts.) True or False.
Make a table in your answer sheet as shown below and fill T (True) or F (false) in the corresponding blank. **必須在答案卷畫出以下表格並在表格內填寫答案.**
每小題2分，未做答0分，答錯一題倒扣2分，最多扣到本大題0分止。

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)

T F (1) The standard error of a statistic is the standard deviation of the statistic.

T F (2) When choosing from a set of possible point estimators, a reasonable approach is to choose the estimator with the smallest mean squared error.

T F (3) We call $\hat{\Theta}$ an unbiased estimator of θ if $\text{Var}(\hat{\Theta}) = \theta$.

T F (4) If $\hat{\Theta}$ is unbiased estimator of θ, then the mean squared error of $\hat{\Theta}$ is identical to the variance of $\hat{\Theta}$.

T F (5) The mean squared error of $\hat{\Theta}$ is defined by $\mathrm{E}(\hat{\Theta} - \mu^2_{\hat{\Theta}})$.

T F (6) If $X_1, \ldots, X_n$ are independent identically distributed (i.i.d.) random variables, then the sample mean $\overline{X}_n = \sum_{i=1}^n X_i/n$ follows a normal distribution for any n.

T F (7) All estimators are statistics, but some statistics are not estimators.

T F (8) Problem (h.1) is a inferential statistics problem and Problem (h.2) is a probability problem.

(h.1) A company can only tolerate 5% defective rate for its manufacturing processes. Suppose that 200 items are inspected and 20 are found to be defective, should we accept the process?

(h.2) If the defective rate is 5%, what is the probability that 200 items are inspected and 20 or more are found to be defective?

T F (9) Applying central limit theorem, the normal distribution can be used to approximate the sampling distribution of the sample mean from a random sample with a large sample size.

T F (10) Applying central limit theorem, normality of the random samples needs to be assumed.

T F (11) Suppose that T follows a t distribution with degrees of freedom 10 and Z follows a standard normal distribution. $\mathrm{P}(T > 2) \geq \mathrm{P}(Z > 2)$.

T F (12) If X_1, X_2, ..., X_n follow a normal distribution with mean μ and variance σ^2, it is also called a random sample.

T F (13) Suppose that we obtain a 95% confidence of the mean μ to be (65.5, 68.4). We know that the unknown mean satisfies $65.5 \leq \mu \leq 68.4$.

T F (14) Suppose that we obtain a 95% confidence of the mean μ to be (65.5, 68.4). We know that $\mathrm{P}(65.5 \leq \mu \leq 68.4) = 0.95$.

T F (15) Suppose that we obtain a 95% confidence of the proportion p to be (0.5, 0.6). We know that if we repeat the experiment many times, in the long run only 95% of the intervals would include the unknown proportion p, but we don't know whether (0.5, 0.6) includes p or not.

T F (16) The life in hours of a battery is known to be approximately normally distributed, with standard deviation $\sigma = 2$ hours. A claim is made that the battery life exceeds 50 hours. Applying the testing-hypothesis approach to make a conclusion, reasonable statistical hypotheses are H_0: $\overline{X} = 50$, H_1: $\overline{X} > 50$, where $\overline{X}$ is the sample mean.

T F (17) For any random variables X and Y, the following equation is true.
$\mathrm{E}(Y) = \mathrm{E}\left(\mathrm{E}(Y|X)\right)$

T F (18) For any random variables X and Y, the following equation is true.
$\mathrm{V}(Y) = \mathrm{V}\left(\mathrm{E}(Y|X)\right)$

T F (19) Suppose that voters, choosing between a Republican and a Democratic candidate, give the Republican $p \times 100\%$ of the votes. We take a random sample of all voters. Consider the statistical hypotheses: H_0: $p \le 0.5$, H_1: $p > 0.5$. Suppose that we take a sample of size n and make a decision that we reject the null hypothesis. That means that the testing-hypothesis approach has helped us to see the truth: "Republican will win".

T F (20) Consider the two-sided test for the ratio of two variances: H_0: $\sigma_1^2/\sigma_2^2 = 1$, H_1: $\sigma_1^2/\sigma_2^2 \neq 1$. If the testing-hypothesis results show that we fail to reject H_0, then $\sigma_1^2 = \sigma_2^2$.

T F (21) Sum of two exponential random variable is also an exponential random variable.

T F (22) $E(X^2)=E(X) \cdot E(X)$, where X is any random variable.

T F (23) The mean and variance of any random variable X are random variables.

T F (24) A gamma random variable is a special case of an exponential random variable.

T F (25) A Bernoulli random variable is a special case of a binomial random variable.

T F (26) Memoryless property $P(X > m+n|X > n)=P(X > m)$ holds for any random variable X.

T F (27) Suppose $\{X(t), t \ge 0\}$ is a Poisson process. $\{X(t_2) - X(t_1)\}$ and $\{X(t_3) - X(t_2)\}$ are independent for all $t_3 > t_2 > t_1$.

T F (28) If $X \sim$ exponential with $E(X) = 10$, then $f_X(x) = 10e^{-10}$.

T F (29) If $X \sim$ Normal distribution with mean μ and variance σ^2, then μ equals to the median of X.

T F (30) Suppose X is a non-negative random variable. Then, Markov Inequality holds: $P(X \ge a) \le E(X)/a$ for all $a > 0$.