

# 國 立 清 華 大 學 命 題 紙

96 學年度工業工程與工程管理學系 (所) 甲組 碩士班入學考試

科目 作業研究 科目代碼 1502 共 5 頁第 1 頁 \*請在【答案卷】內作答

- 注意事項: (1) 不得使用計算器。  
 (2) 請依題號順序作答。  
 (3) 答案必須寫在答案卷上, 並須依每一題規定的方式作答。  
 (4) 未依規定作答, 酌量扣分。

10%

1. True (T) or False (F)? 以下是非題每一小題 2 分, 但答錯一小題倒扣 2 分, 最多只扣到本題 0 分, 必須在答案卷畫出以下表格並在表格內填寫答案。

題目 1	(1-1)	(1-2)	(1-3)	(1-4)	(1-5)
答案					

- (1-1) In two dimensional LP solution, the objective function can assume the same value at two distinct extreme points.
- (1-2) Every two-person zero-sum game with pure strategies can be represented by a pair of primal-dual linear programs.
- (1-3) The maximum flow between the two nodes in a network may exceed the capacity of its minimal cut.
- (1-4) Every basic solution in the assignment problem is necessarily degenerate.
- (1-5) The revised simplex method is designed to reduce the amount of computer memory, but it has the same amount of computation as the simplex method.

10%

2. 本題必須畫圖在答案卷上, 推導後在答案卷畫出以下表格並在表格內填寫答案。

題目 2 答案	
optimal solution	range of $k$
$(x_1, x_2) = (0, 5)$	
$(x_1, x_2) = (2, 4)$	
$(x_1, x_2) = (6, 0)$	

Consider the following problem, where the value of  $k$  has not yet been ascertained.

$$\begin{aligned}
 \text{(P)} \quad & \text{Maximize} \quad z = kx_1 + x_2 \\
 & \text{subject to} \quad x_1 + x_2 \leq 6 \\
 & \quad \quad \quad x_1 + 2x_2 \leq 10, \quad x_j \geq 0, \quad j = 1, 2.
 \end{aligned}$$

(P) has four extreme points  $(0, 0)$ ,  $(0, 5)$ ,  $(2, 4)$  and  $(6, 0)$ . Using graphical analysis to determine the optimal solution(s) for  $(x_1, x_2)$  for various possible values of  $k$  ( $-\infty < k < \infty$ ).

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10%

3. 本題推導過程必須寫在答案卷上, 推導後在答案卷畫出以下表格並在表格內填寫答案。

題目 3	答案
目標式 (objective function)	
Maximize or Minimize	
條件式 (constraints)	
變數 $x_j$ 之限制式 (constraints for variables)	

Only one of the following problems can be formulated a linear program. Find it and formulate it as a linear program with equality or inequality constraints.

(a)

$$\text{Maximize } z = c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 x_4$$

$$\text{subject to } a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \leq b_2$$

$$x_j \geq 0, \quad j = 1, 2, 3, 4.$$

} At least one of the two inequalities holds.

(b). Minimize maximize  $\{ |c_1 x_1 + c_2 x_2 - A|, |d_1 x_1 + d_2 x_2 - B| \}$

$$\text{subject to } x_j \geq 0, \quad j = 1, 2.$$

(c) Maximize

$$c_1 x_1 + c_2 x_2$$

subject to

$$a_{11} x_1 + a_{12} x_2 \leq b_1$$

$$a_{21} x_1 + a_{22} x_2 \leq b_2$$

$$|x_1 - x_2| = 0, 3, \text{ or } 6.$$

$$x_j \geq 0, \quad j = 1, 2.$$

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20%

4. 本題計算過程與推導必須寫在答案卷上, 計算後必須在答案卷畫出以下表格並在表格內填寫答案。

題目 4	答案
(4-1)	
(4-2)	
(4-3)	
(4-4)	
(4-5)	
(4-6)	range of $t$ :

The linear program

$$\begin{aligned} \text{Maximize } z &= 3x_1 + 6x_2 \\ \text{subject to } x_1 &\leq 4 \\ 3x_1 + 2x_2 &\leq 18 \\ x_j &\geq 0, \quad j = 1, 2, \end{aligned}$$

has the final simplex tableau

Basic Variable	$z$	$x_1$	$x_2$	$x_3$	$x_4$	Right Side
$z$	1	6	0	0	1	54
$x_3$	0	1	0	1	0	4
$x_2$	0	3/2	1	0	1/2	9

where  $x_3$  and  $x_4$  are slack variables.

Consider the following linear parametric programming problem.

$$\begin{aligned} \text{Maximize } z &= 3x_1 + (6 - 4t)x_2 \\ \text{subject to } (1 + 2t)x_1 &\leq 4 + 8t \\ (3 - 3t)x_1 + 2x_2 &\leq 18 - 24t \\ x_j &\geq 0, \quad j = 1, 2, \end{aligned}$$

where  $t \geq 0$ . Find (4-1), (4-2), (4-3), (4-4), and (4-5) (as functions of  $t$ ) in the following resulting revised final tableau. ((4-1) – (4-5) 3% each)

Basic Variable	$z$	$x_1$	$x_2$	$x_3$	$x_4$	Right Side
$z$	1	(4-1)			(4-2)	(4-3)
$x_3$	0					(4-4)
$x_2$	0					(4-5)

Give the range of  $t$  for which the current basic solution is both feasible and optimal. (4-6) 5%

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15%

5. 以下選擇題(單選題)每一小題 3 分, 共 15 分, 必須在答案卷畫出以下表格並在表格內填寫答案。

題目 5	(5-1)	(5-2)	(5-3)	(5-4)	(5-5)
答案					

- (5-1) The goal of queueing analysis is to minimize:  
(A) the sum of customer waiting costs and capacity costs (B) the sum of customer waiting time and service time (C) capacity costs (D) customer waiting time (E) none of the above
- (5-2) The term “queue discipline” refers to:  
(A) the willingness of customers to wait in line for service (B) having multiple waiting lines without customers switching from line to line (C) the order in which customers are processed (D) the reason waiting occurs in underloaded systems (E) none of these
- (5-3) As the ratio of arrival rate to service rate is increased, which of the following is likely?  
(A) Customers move through the system in less time because utilization is increased.  
(B) Customers move through the system more slowly because utilization is increased.  
(C) Utilization is decreased because of the added strain on the system. (D) The average number in the system decreases. (E) None of the above will occur.
- (5-4) The basic priority-discipline queueing model assumes that:  
(A) arrival rates are exponentially distributed (B) service times are Poisson distributed  
(C) items are services in order of arrival (D) items are serviced in order of priority class  
(E) service activities are preemptive
- (5-5) A multiple channel system has customers arriving at an average rate of five per hour and an average service time of forty minutes. The minimum number of servers for this system to be underloaded is:  
(A) 2 (B) 3 (C) 4 (D) 5 (E) none of these

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15%

6. 本題計算過程與推導必須寫在答案卷上, 計算後必須在答案卷畫出以下表格並在表格內填寫答案。

題目 6	答案
(6-1)	
(6-2)	
(6-3)	

Repair calls are handled by one repairman at a photocopy shop. Repair time, including travel time, is exponentially distributed, with a mean of two hours per call during an eight-hour day. Requests for copier repairs come in at a mean rate of three per eight-hour day (assume Poisson). Determine:

(6-1) The average number of customers awaiting repairs. (5 分)

(6-2) The amount of time during an eight-hour day that the repairman is not out on a call. (5 分)

(6-3) The probability of two or more customers in the system. (5 分)

20%

7. 本題計算過程與推導必須寫在答案卷上, 計算後必須在答案卷畫出以下表格並在表格內填寫答案。

題目 7 答案	

Trucks arrive at a warehouse according to a Poisson process with a mean rate of 4 per hour. Only one truck can be loaded at a time. The time required to load a truck has an exponential distribution with a mean of  $10/n$  minutes, where  $n$  is the number of loaders ( $n = 1, 2, 3, \dots$ ). The cost are (i) \$18 per hour for each loader and (ii) \$20 per hour for each truck being loaded or waiting in line to be loaded. Determine the number of loaders that minimizes the expected hourly cost.