國　立　清　華　大　學　命　題　紙

96 學年度工業工程與工程管理學系（所）甲、乙、丙組 碩士班入學考試

科目 統計學 科目代碼 1501、1601、1701 共 3 頁第 1 頁 *請在【答案卷卡】內作答

註：不得使用計算器。

Problem 1.

It is known that any item produced by a certain machine will be defective with probability p, independently of any other item. Let X be the number of defective parts in a sample of n items.

(a) (10%) Show that as k goes from 0 to n, $Pr(X = k)$ increases monotonically, then decreases monotonically.

(b) (10%) In part (a), when will $Pr(X = k)$ reach its largest value?

(c) (8%) Show that $Var(X) = np(1 - p)$. ($Var(X)$ is the variance of X.)

(d) (7%) Let X_1 be the number of defective parts in a sample of 5 items and X_2 be the number of defective parts in another sample of 3 items. Calculate the conditional probability mass function $Pr(X_1 = k | X_1 + X_2 = 2)$.

Problem 2.

(8%) Given $\overline{X} = 20$ for a random sample size of 25 from the density $f(x|\mu)=\frac{1}{4\sqrt{2\pi}} e^{-[(x-\mu)^2/32]}$, find a 90% confidence interval for μ.

Problem 3.

(9%) Determine the nature of the best critical region, based on a sample of size n, for testing the hypothesis

$$H_0 : \theta = \theta_0 \text{ against } H_1 : \theta = \theta_1 < \theta_0$$

if the probability density function $f(x|\theta)=\theta e^{-\theta x}$, $x > 0$, and $\theta > 0$.

Problem 4.

(8%) A coin is tossed 900 times. The owner of the coin alleges that it is biased. How many heads must turn up in 900 tosses for $p = 0.5$ to be rejected at the 5% level of significance?

Problem 5.

(10%) Given $H_0 : \mu = \mu_0$

$H_1 : \mu = \mu_1$,

and α and β are possibilities of making type I and type II errors respectively, show that for a two-tailed test the required sample size n is given approximately by

$$n = \frac{(z_{\alpha/2} + z_{\beta})^2 \sigma^2}{(\mu_1 - \mu_0)^2},$$

provided that

$$Pr(z < -z_{\alpha/2} - \frac{\sqrt{n}|\mu_1 - \mu_0|}{\sigma})$$

is small when $\mu = \mu_1$.

Note that the most commonly used confidence coefficients with corresponding values of α and $z_{\alpha/2}$ are shown in the following table.

confidence interval 100(1-α)	α	$z_{\alpha/2}$
90%	0.10	1.645
95%	0.05	1.960
99%	0.01	2.575

Problem 6.

Ten observations on the yield of an operation (y) and one process variable-reaction time (x) are collected. We fit a simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$ to the data. Here is part of the Minitab output for this regression:

```
The regression equation is
Yield = 10.0 + 2.0 time

Analysis of Variance
Source              DF        SS        MS         F
Regression           1      1800      1800       200
Residual Error
Total                9      1872
```

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註：不得使用計算器。

(a) (3%) Complete the ANOVA table by filling in the "Residual Error" row.

(b) (3%) Compute the value of the regression standard error (s).

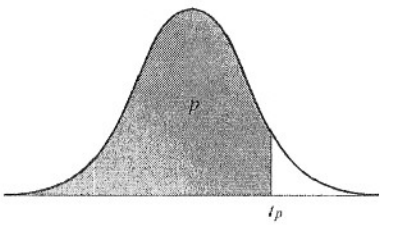
(c) (5%) Compute the value of the squared correlation (r^2).

(d) (5%) The standard deviation of the x is 20.0. Find the standard error for the least-squares slope (b_1).

(e) (7%) Compute the test statistic for $H_0 : \beta_1 = 0$ and describe your conclusion ($\alpha = 0.05$).

(f) (7%) Give a 95% confidence interval for the slope β_1 of the regression line.

t distribution critical values



df	$t_{.75}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$
1	1.000	3.078	6.314	12.71	31.82	63.66	318.3
2	0.816	1.886	2.920	4.303	6.965	9.925	22.33
3	0.765	1.638	2.353	3.182	4.541	5.841	10.21
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090