不得使用計算器

注意事項: (1) 請依題號順序作答。
(2) 答案必須寫在答案卷上,計算過程與推導可寫在答案卷上,但答案須依每一題規定的方式作答。

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1. True (T) or False (F)? 是非題每一小題 2 分, 但答錯一小題倒扣 2 分, 最多只扣到本題 0 分, 必須在答案卷畫出以下表格並在表格內填寫答案。

題目	(1-1)	(1-2)	(1-3)	(1-4)	(1-5)	(1-6)	(1-7)	(1-8)	(1-9)	(1-10)
答案										

(1-1) If a waiting customer becomes impatient, he may decide to *renege*.

(1-2) *Jockeying* is exercised by customers in a single-server facility in the hope of reducing their waiting time.

(1-3) In a queuing system, if the arrivals occur according to a Poisson distribution, the interarrival time is exponential distributed.

(1-4) The mean and variance of the Poisson distribution may not be equal.

(1-5) Under the Poisson assumption, two arrivals can occur during a very small time interval.

(1-6) An arriving customer may *balk* if he expects a long waiting time.

(1-7) If the time between successive arrivals is exponential, the time between the occurrence of every third arrival is also exponential.

(1-8) The arrival rate in the Poisson distribution equals the mean of the exponential interarrival time.

(1-9) In a single-server queuing system, steady state can be reached after a sufficiently long period only if the arrival rate is less than the service rate.

(1-10) In the preemptive priority queues, a service can be interrupted in favor of an arriving customer with higher priority.

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2. (2-1) Consider the following one-step transition matrix of a Markov chain (with states 0, 1, 2, 3, 4).

$$
\begin{array}{c} \text{States} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix}
\frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\
\frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} \\
0 & 0 & 0 & \frac{1}{4} & \frac{3}{4}
\end{bmatrix} \end{array}
$$

Determine the classes of the Markov chain and whether they are recurrent.

(2-2) Consider an $M/M/1$ system with mean arrival rate λ and service rate μ,

Give $P\{W_q > t\}$ for $t \geq 0$. (必須有推導過程,提示 $P\{W > t\} = e^{-\mu(1-\rho)t}$, $\rho = \lambda/\mu$, $t \geq 0$。)

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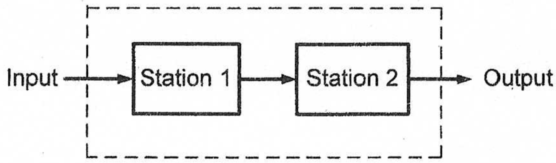
3. Consider a queuing system consisting of two series stations as shown below.



A customer arriving for service must go through station 1 and station 2. Service time at each station are exponential distributed with the same service rate μ. Arrivals occur according to a Poisson distribution with rate λ. No queues are allowed in front of station 1 and station 2. Each station may be either **free** or **busy**. Station 1 is said to be **blocked** if the customer in this station completes his or her service before station 2 becomes free. In this case, the customer cannot wait between the stations since this is not allowed. Let 0, 1, and b represent the **free**, **busy**, and **blocked** states, respectively. Let i and j represent the states of states of station 1 and station 2. Then the states of the system are given by

$$\{(i, j)\} = \{(0, 0), (0, 1), (1, 0), (1, 1), (b, 1)\}.$$

Let $p_{ij}(t)$ be the probability that the system is in state (i, j) at time t.

(3-1) Give the transition probabilities between times t and $t + h$ (h is a small positive increment in time.)

(3-2) Give the steady-state equations.

(3-3) Solve p_{00}, p_{01}, p_{10}, p_{11}, p_{b1}.

(3-4) Find the expected number in the system L_s.

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4. Consider a maximization linear programming problem

(4-1) Write down this model with specific dimensions.

(4-2) State the Simplex Algorithm and estimate its complexity.

(4-3) State the possibilities of the feasible solutions with the respective conditions.

(4-4) State the possibilities of the optimal solutions with the respective conditions.

(4-5) State how to calibrate the model if the model has

(a).no feasible solution; (b) no optimal solution.

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5. Giving the maximization LP model as in problem (4-1)

(5-1) Define the respective dual model.

(5-2) State the relation of these two models and provide the evidence.

(5-3) State which model should be used and why in terms of

(a). Efficiency in finding solutions.

(b). Sensitivity Analysis.