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九十三學年度 工業工程與工程管理學系（所）工業工程組甲組碩士班入學考試

科目 作業研究(含線性規劃) 科號 1702 共 6 頁第 1 頁 *請在試卷【答案卷】內作答

注意事項：(1) 請依題號順序作答。

(2) 答案必須寫在答案卷上,計算過程與推導可寫在答案卷上,但答案必須依每一題規定的方式作答。

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1. 應用 dual simplex method 解以下問題:

$$\begin{aligned} \textit{maximize} \quad & Z = -2x_1 - 2x_2 \\ \textit{subject to} \quad & 2x_1 + x_2 \geq 6 \\ & x_1 + 2x_2 \geq 6, \quad x_i \geq 0, \quad i = 1, 2. \end{aligned}$$

令 x_3, x_4 為 slack variables, 其 Tableau 1 為

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	Right Side
z	(0)	1	2	2	0	0	0
x_3	(1)	0	–2	–1	1	0	–6
x_4	(2)	0	–1	–2	0	1	–6

畫以下形式的表格求解最佳解(optimal solution)。

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	Right Side
z	(0)	1					
	(1)	0					
	(2)	0					

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2. 考慮以下運輸規劃問題(transportation problem):

		Destination 1	2	3	4	Supply
Source	1	10	2	20	11	15
	2	12	7	9	20	25
	3	4	14	16	18	10
Demand		5	15	20	10	

如 transportation simplex method 演算到第 k 階段(the k-th iteration), 其 basic variables 為 $x_{12}, x_{14}, x_{21}, x_{22}, x_{23}, x_{34}$ (a)求解 $x_{12}, x_{14}, x_{21}, x_{22}, x_{23}, x_{34}$ (b) 令 $u_1 = 0$ 並求解其它的 dual variables u_i, v_j, $i = 2, 3$, $j = 1, 2, 3, 4$。(c)求解所有 nonbasic variables 的 reduced cost λ_{ij}。

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科目 [illegible] 科號 [illegible] 共 6 頁第 2 頁 *請在試卷【答案卷】內作答

在答案卷畫出以下表格並在表格內填寫(a) (b) (c)的答案。

	1	2	3	4	u_i
1	$\lambda_{11}=$	$x_{12}=$	$\lambda_{13}=$	$x_{14}=$	$u_1=0$
2	$x_{21}=$	$x_{22}=$	$x_{23}=$	$\lambda_{24}=$	$u_2=$
3	$\lambda_{31}=$	$\lambda_{32}=$	$\lambda_{33}=$	$x_{34}=$	$u_3=$
v_j	$v_1=$	$v_2=$	$v_3=$	$v_4=$	

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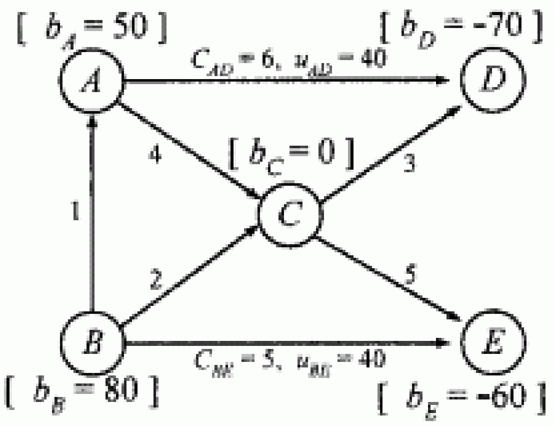
3. 考慮以下的 minimum cost flow problem:



$C_{AC} = 4$, $u_{AC} = +\infty$　　$C_{BE} = 5$, $u_{BE} = 40$

$C_{AD} = 6$, $u_{AD} = 40$　　$C_{CD} = 3$, $u_{CD} = +\infty$

$C_{BA} = 1$, $u_{BA} = +\infty$　　$C_{CE} = 5$, $u_{CE} = +\infty$

$C_{BC} = 2$, $u_{BC} = +\infty$

C_{ij} = cost per unit flow through arc $i \to j$.

u_{ij} = arc capacity for arc $i \to j$.

b_i = net flow generated at node i.

x_{ij} = flow through arc $i \to j$.

(3-1) 寫出此問題的 linear programming formulation. 在答案卷畫出以下表格並在表格內填寫答案。

objective function	
constraint on node A	
constraint on node B	
constraint on node C	
constraint on node D	
constraint on node E	
upper bounds for x_{ij}	

(3-2) 若 $x_{AC}, x_{BA}, x_{CD}, x_{CE}$, 為 basic variables 且 $x_{AD}= 40$, $x_{BC} = x_{BE} = 0$, (a)求解 $x_{AC}, x_{BA}, x_{CD}, x_{CE}$ (b) 求解 nonbasic variables 的 reduced costs $\lambda_{AD}, \lambda_{BC}, \lambda_{BE}$. 在答案卷畫出以下表格並在表格內填寫答案。

x_{AC} =	x_{BA} =	x_{CD} =	x_{CE} =
λ_{AD} =	λ_{BC} =	λ_{BE} =	

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4. 假如有一套裝軟體(software package)可解以下樣式的聯立方程式:

$$\mathbf{Dz} \le \mathbf{g}, \quad \mathbf{z} \ge \mathbf{0}.$$

$$\mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_m \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$$

$(-\infty < d_{ij} < +\infty,\ -\infty < g_i < +\infty,\ i = 1, 2, \ldots, m,\quad j = 1, 2, \ldots, n.)$

請問此軟體是否可用來解以下線性規劃問題(Linear Program)? 必須說明或證明您的答覆。

$$\begin{aligned} &\textit{maximize} \quad \mathbf{cx} \\ &\textit{subject to} \quad \mathbf{Ax} \le \mathbf{b}, \quad \mathbf{x} \ge \mathbf{0} \end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1h} \\ a_{21} & a_{22} & \cdots & a_{2h} \\ \vdots & \vdots & \cdots & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{kh} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_h \end{bmatrix}$$

$(-\infty < a_{ij} < +\infty,\ -\infty < b_i < +\infty,\ i = 1, 2, \ldots, k,\quad j = 1, 2, \ldots, h.)$

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5. How to analyze the output process of two M/M/1 queues in serious (Each customer must first be served by the first queue and then served by the second queue before leaving the system) on a FCFS basis with an arrival rate of λ_1 and a service rate of μ_1 for the first queue and a service rate of μ_2 for the second queue?

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6. If a store opens at 9:30 A.M., time 0 would be 9:30 and A(t) would be the number of customers who arrived between 9:30 and time t. Consider the following data:

Customer	Arrival time	Departure from queue	Departure from system
1	9:36	9:36	9:40
2	9:37	9:40	9:44
3	9:38	9:44	9:48
4	9:40	9:48	9:52
5	9:45	9:52	9:56

Let $A(t)$ = Cumulative arrivals from time 0 to time t

$Ds(t)$ = Cumulative departures from the system from time 0 to time t

$Dq(t)$ = Cumulative departures from the queue from time 0 to time t

$A^{-1}(n)$ = time of the nth arrival

$D_q^{-1}(n)$ = time of the nth departure from queue

$D_s^{-1}(n)$ = time of the nth departure from system

a = start time of interval

b = end time of interval

$L_q(t)$ = number of customers in the queue at time t

L_q = average queue length (customers)

σ_{wq} = standard deviation of time in queue

σ_{ws} = standard deviation of time in system

σ_{Lq} = standard deviation of queue length

σ_{Ls} = standard deviation of customers in the system

Ls = average number of customers in the system

$L_s(t)$ = number of customers in the system at time t

(1) Find an expression for Lq(t), the number of customers in the queue at time t, and Ls(t), the number of customers in the system at time t.

(2) Find an expression for Wq(n), time in queue, for nth customer to arrive, and Ws(n), time in system, for nth customer to arrive.

(3) Find an expression for Wq, average waiting time in queue and calculate the number for the above example. Find an expression for Ws, average waiting time in system and calculate the number for the above example.

(4) Find an expression for σ_{wq}, the standard deviation of time in queue and plug in the data of the example into the expression (You don't need to calculate the number.). Find an expression for σ_{ws}, the standard deviation of time in system and plug in the data of the example into the expression (You don't need to calculate the number.).

(5) Find an expression for Lq and σ_{Lq}.

(6) Find an expression for Ls and σ_{Ls}.

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7. A cargo plane has three compartments for storing cargo: front, center, and back. The compartments have capacity limits on both weight and space, as summarized below:

Compartment	Weight Capacity (Tons)	Space Capacity (Cubic Feet)
Front	13	7,000
Center	17	9,000
Back	10	5,000

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the airplane.

The following four cargos have been offered for shipment on an upcoming flight as space is available:

Cargo	Weight (Tons)	Volume (Cubic Feet/Ton)	Profit ($/Ton)
1	20	500	330
2	16	700	400
3	24	600	360
4	13	400	290

Any portion of these cargos can be accepted. The objective is to determine how much (if any) of each cargo should be accepted and how to distribute each among the compartments to maximize the profit of the flight.

(1) Formulate the linear programming model for this problem.

(2) Try to set up an initial iteration of the simplex method for the problem, and then carry out one iteration.

(3) Write a dual problem for Problem 1 and list three possible applications of the dual problem.

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8. Four cargo ships will be used for shipping goods from one port to four other ports (labeled 1,2,3,4). Any ship can be used for making any one of these four trips. However, because of differences in the ships and cargos, the loading, transporting, and unloading the goods for the different ship-port combinations varies considerably, as shown in the following table:

		Port 1	Port 2	Port 3	Port 4
Ship	1	5	4	6	7
	2	6	6	7	5
	3	7	5	7	6
	4	5	4	6	6

The objective is to assign the ships to ports on a one-to-one basis in such a way to minimize the total cost for all four shipments.

(1) Describe how this problem fits into the general format for the assignment problem.

(2) Reformulate this problem as an equivalent transportation problem by constructing the appropriate cost and requirements table.

(3) Use the northwest corner rule to obtain an initial basic feasible solution for the problem as formulated in part (b).