1. (15 pts.) Let $\bar{X} = \sum_{i=1}^{n} X_i/n$, where $X_i, i = 1, ..., n$ are a random sample of size n for a normal distribution with unknown mean μ_X and known variance σ_X.

   (a) Prove or disprove that there exists some constants a and b such that

   $$a\bar{X} + b \tag{1}$$

   is a standard normal random variable.

   (b) Explain how Equation (1) can be used for estimating the unknown parameter μ.

2. (15 pts.) Let $\bar{X} = \sum_{i=1}^{n} X_i/n$, where $X_i, i = 1, ..., n$ are a random sample of size n for a normal distribution with unknown mean μ_X and unknown variance σ_X.

   (a) Prove or disprove that there exists some constants a, b, c such that

   $$a\bar{X} + b \tag{2}$$

   has a t distribution (student distribution) with degrees of freedom c.

   (b) Explain how Equation (2) can be used for estimating the unknown parameter μ.

3. (20 pts.) Below are four mathematicians who made important contributions on the area of probability or statistics. Regarding to each person below, name one of their contribution on the area of probability or statistics. (Please do no just give the result or the name of theorem. Interpret the result or theorem including how such result or theorem is useful)

   (a) Jacob (Jacques) Bernoulli (1654-1705) (Switzerland)

   (b) Thomas Bayes (1702-1761) (England)

   (c) Johann Carl Friedrich Gauss (1777-1855) (German)

   (d) Pafnuty Lvovich Chebyshev (1821-1894) (Russian)

4. (25 pts.) Data mining is an approach that uses statistical methods and data analysis techniques to extra valuable information from large amount of data so as to support business decision. One commercial bank recently issues a "student cash card" that allows users to borrow money from the bank up to NT$200,000 with annual interest rate 18%. After promotion, they have received 600,000 approved applicants. After one year, they found that 55% of customers never really use this card to borrow money. 15% of the customers have borrowed all the money of NT$200,000, yet failed to pay the interest and return the money. Only 30% of the customers have borrowed the money and still maintain good credits. Please describe several statistics (and/or data mining) approaches to extra potentially useful information you want and the corresponding methods.

5. (25 pts.) What is meaning of "unbiased estimator" in Statistics? Please define it and/or explain it with examples. Furthermore, given the following table, there is one missing value in 11 data items of one variable. Suppose you want to replace the missing value with an estimated value. If "sample mean" is an unbiased estimator, what value can we replace the missing value with? If "standard deviation" is an unbiased estimator, what value can we replace the missing value with? Give your reasons and/or calculation process.

No.	Variable 1
1	.0086
2	.0684
3	.3515
4	.9874
5	.4723
6	.6115
7	.2573
8	Missing
9	.2914
10	.1662
11	.4400