1. 以下每小題各 4 分共 20%

1-1 For the products a, b, c, d, which of the following could be a linear programming objective function?

(A) z = 1a + 2b + 3c + 4d

(B) z = 1a + 2bc + 3d

(C) z = 1a + 2ab + 3abc + 4abcd

(D) z = 1a + 2b/c + 3d

(E) all of the above

1-2 What combination of x and y will yield the optimum for the problem?

Maximize z = 3x + 15y

Subject to :

$2x + 4y \leq 12$

$5x + 2y \leq 10$

(A) x = 2, y = 0

(B) x = 0, y = 0

(C) x = 0, y = 3

(D) x = 1, y = 5

(E) none of the above

1-3 In graphical linear programming, when the objective function is parallel to one of the constraints, then:

(A) the solution is suboptimal

(B) multiple optimal solutions exist

(C) a single corner point solution exists

(D) no feasible solution exists

(E) none of the above

1-4 A shadow price reflects which of the following in a maximization problem?

(A) the marginal cost of adding additional resources

(B) the marginal gain in the objective that would be realized by adding one unit of a resource

(C) the net gain in the objective that would be realized by adding one unit of a resource

(D) the marginal gain in the objective that would be realized by subtracting one unit of a resource

(E) none of the above

1-5 In linear programming, sensitivity analysis is associated with:

I. objective function coefficient

II. right-hand side values of constraints

III. constraint coefficient

(A) I and II

(B) II and III

(C) I, II and III

(D) I and III

(E) None of the above

20%

2. The following is the current simplex tableau of a given maximization problem. The objective is to maximize $2x_1 - 3x_2$, and the slack variables are x_3 and x_4. The constraints are of the ≤ type.

basis	z	x_1	x_2	x_3	x_4	RHS
z	1	***b***	1	***f***	***g***	6
x_3	0	***c***	0	1	1/5	4
x_1	0	***d***	***e***	0	2	***a***

(1) Find the unknowns ***a*** through ***g***. (14%)

(2) Is the tableau optimal? (3%)

(3) Does there exist alternative optimal solution? (3%)

10%

3. Consider the following LP:

Minimize $z = 2x_1 + 15x_2 + 5x_3 + 6x_4$

subject to.

$$x_1 + 6x_2 + 3x_3 + x_4 \geq 2$$

$$2x_1 - 5x_2 + x_3 - 3x_4 \geq 3$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$

(1) Give the dual of the problem. (5%)

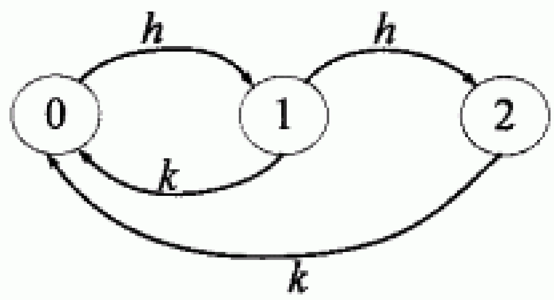
(2) Solve the dual geometrically. (5%)

5%

4. For $M/M/1/1$ system (λ = average customer arrival rate, μ = average service rate), find the steady-state probability P_k = Prob(k customers in system) for each k.

10%

5. Consider the Markovian queueing system with the following state transition diagram. Find the average number of customers in the system.



10%

6. (6 - *a*) Explain the "memoryless property".

   (6 - *b*) Give one continuous-time Markov chain which has memoryless property and prove your answer.

15%

7. In an $M/M/1$ system (λ = average customer arrival rate, μ = average service rate), we assume that the probability of a customer's balking is $1-(1/2)^n$ where n is the number of customers in the system.

   (7 - *a*) Construct the state transition diagram for this process.

   (7 - *b*) Develop the balance equations.

   (7 - *c*) Solve for steady-state probabilities P_k (in terms of P_0) for each $k \geq 1$.

10%

8. Consider an $M/M/1$ system where λ is 0.25 arrivals per minute. Let C_r be the cost for serving one unit and C_w be the cost of waiting per unit per minute. If C_r = \$0.04 and C_w = \$0.05, then find the ***minimum cost service rate***.