

國立清華大學命題紙

九十一學年度 工業工程與工程管理學系 (所) 甲, 乙, 丙 組碩士班研究生招生考試

科目 統計學 科號 1701 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (10 pts.) Consider the function below

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ x - \frac{1}{4}x^2 & \text{if } 0 \leq x \leq 2 \\ 1 & \text{if } 2 \leq x \end{cases}$$

Is the above function a cumulative density function (cdf) of a random variable X? Why or why not. If it is a cdf of X, determine the corresponding probability density function (pdf) of X.

2. (10 pts.) Yearly demand on a supplier for a given item has been 10 in the past, and demands seem to be distributed as a negative binomial random variable with parameters $r = 2$ and $p = 0.2$. An inventory manager stocks 8 items for the coming year. What is the probability that he will not be able to supply the demand this year?
3. (15 pts.) The grades for 15 students on midterm and final examinations in a Statistics course are given in the following table:

Midterm(x)	Final(y)	
82	76	
73	83	
95	89	
66	76	
84	79	
89	73	
51	62	
82	89	$\Sigma x = 1,098$
75	77	$\Sigma y = 1,056$
90	85	$\Sigma x^2 = 84,748$
60	48	$\Sigma y^2 = 78,618$
81	69	$\bar{x} = 73.2$
34	51	$\bar{y} = 70.4$
49	25	$\Sigma xy = 80,646$
87	74	

- (a) Obtain the least squares regression to predict the score on the final examination from the midterm examination score. Test for significance of the regression and interpret the results.
- (b) Plot the estimated line and the actual data points. Comment on these results.
- (c) Predict the final score for a student who made a score of 80 on the midterm.
4. (15 pts.) To assess the effectiveness of a new diet formulation, a sample of eight steers is fed a regular diet and another sample of ten steers is fed a new diet. The weights of the steers at 1 year are given in the following table. Do these results imply that the new diet results in higher weights? (Use $\alpha = 0.05$).

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Regular Diet	New Diet
831	870
858	882
833	896
860	925
922	842
875	908
797	944
788	927
	965
	887
Average = 845.5 $S^2 = 1,873$	Average = 904.6 $S^2 = 1,349$

5. (24 pts.) Let X_1, X_2, \dots, X_n be a random sample of size n from a normal distribution with mean μ_X and variance σ_X^2 . Let S_X be the sample standard deviation of X . Consider the following six statistics
- $(X_1 + X_2)/2$
 - \bar{X}_n
 - $\bar{X}_n - \mu_X$
 - $(\bar{X}_n - \mu_X)/(\sigma_X/\sqrt{n})$
 - $(\bar{X}_n - \mu_X)/(S_X/\sqrt{n})$
 - $\sum_{i=1}^n (X_i - \bar{X})^2/\sigma_X^2$
- (a) Write down the name of the distribution including the corresponding parameters of the above six statistics. Make any assumptions if necessary.
- (b) Let the observed data of X_1, X_2, \dots, X_5 be 2, 4, 6, 8, 10. Draw the empirical probability density function (pdf) of the above six statistics.
6. (14 pts.) Let X_1, X_2, \dots, X_n be a random sample of size n from a normal distribution with mean μ_X and variance σ_X^2 . Prove or disapprove that $\sum_{i=1}^n (X_i - \mu_X)^2/\sigma_X^2$ follows a chi-squared distribution with degrees of freedom n .
7. (12 pts.) Suppose one has a stereo system consisting of two main parts, a radio and a speaker. If the lifetime of the radio is exponential with mean 1000 hours and the lifetime of the speaker is exponential with mean 500 hours independent of the radio's lifetime, then what is the probability that the systems' failure (when it occurs) will be caused by the radio failing?