

可使用非程式型(不具儲存功能)計算機

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1. (1-a) For an $M/M/1$ system, does $L_q = L - 1$?(1-b) In the $M/M/s/K$ queueing system, do we need the condition $\lambda < \mu$?

You should give the reasons for your answers to (1-1) and (1-2). 不說明理由, 將不予計分。

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2. Consider the *birth-and-death* process with the following mean rates. The birth rate are $\lambda_0=3$, $\lambda_1=4$, $\lambda_2=2$, $\lambda_3=1$, and $\lambda_n=0$ for $n > 3$. The death rate are $\mu_1=2$, $\mu_2=3$, $\mu_3=1$, and $\mu_n=2$ for $n > 3$.

(2-a) Construct the rate diagram for this birth-and-death process.

(2-b) Develop the balance equations.

(2-c) Solve the steady-state probability P_i , $i = 0, 1, \dots$ (2-d) Calculate L and W .

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3. The following observations have been made in a queueing system. There is only *one server* with the first-come, first-served basis. The time between successive arrivals and the actual time to serve them (excluding any waiting time) are given as follows.

Customer number	#1	#2	#3	#4	#5	#6
Interarrival Time	-	9	6	4	7	9
Service Time	3	7	9	9	10	4

(3-a) Calculate the average time a customer must wait before being service.

(3-b) Calculate the average waiting time including service.

(3-c) Calculate the average length of the queue.

(3-d) Calculate the average number of customers in the system.

(3-e) Calculate the total idle time of the server.

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4. Given a linear program as follows :

$$\begin{aligned} \text{Max } z &= 3x_1 + 2x_2 \\ \text{s.t. } 2x_1 + x_2 &\leq 100 \\ x_1 + x_2 &\leq 80 \\ x_1 &\leq 40 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(4-a). (5%) Find optimal solution by graphic method.

(4-b). (5%) For what values of the coefficient of x_1 in objective function does the current basis remain optimal? Explain from the graph.(4-c). (5%) For what values of the right hand side of the 1st constraint does the current basis remain optimal? Explain from the graph.

(4-d). (10%) What is Shadow Price? Analyze shadow prices of the above problem directly without using Simplex Tableau.

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5. Accer is about to introduce a new product (product 3). One unit of product 3 is produced by assembling 1 unit of product 1 and 1 unit of product 2. Before production begins on either product 1 or product 2, raw materials must be purchased and workers must be trained. Before products 1 and 2 can be assembled into product 3, the finished product 2 must be inspected. A list of activities and their predecessors and of the duration of each activity is given in table below.

Activity	Predecessors	Duration(days)
A = train workers	-----	6
B = purchase raw materials	-----	9
C = produce product 1	A, B	8
D = produce product 2	A, B	7
E = test product 2	D	10
F = assemble products 1 & 2	C, E	12

(5-a). (10%) Draw a project diagram for this project with early and late event time.

(5-b). (10%) Finding critical path both from the graph and formulate such LP model.

(5-c). (5%) Since Accer's competitor is scheduled to hit the market 26 days, so Accer must introduce product 3 within 25 days. If the reduced duration of any activity can be up to 5 days with the cost per day shown below. Formulate an LP model to support Accer's decision on how to complete the project by the 25-day deadline with the minimum cost.