(共五題，請依規定將答案寫在答案卷上)

(20%)

1. Yes or No. Justify your answers.

(1-a) If an optimal solution exists for an linear program, then an optimal extreme point exists.

(1-b) For every extreme point of an LP, there corresponds a basis and vice versa.

(1-c) An advantage of the simplex method is that it will always terminate in a finite number of iterations.

(1-d) Once an primal LP is unbounded, we know that its dual is unbounded too and vise versa.

(1-e) When both primal and dual have the same feasible basic solutions, they both reach optimality.

(30%)

2. Consider a polyhedral set defined by the following inequalities

$$\begin{aligned} x_1 + x_2 &\le 6 \\ x_2 &\le 3 \\ x_1 + 2x_2 &\le 9 \\ x_1, \quad x_2 &\ge 0 \end{aligned}$$

(2-a) Find the basic feasible solutions by introducing the slack variables.

(2-b) Show that the number of basic feasible solutions for m constraints and n variables is less than or equal to $\binom{n}{m}$.

(2-c) Draw the figure of (2-a) and explain its result with (2-b).

(2-d) If $2x_1 + x_2$ is going to be maximized, what would be the optimal solution(s) from the figure of (2-c) ?

(2-e) If the resource of 9 has been expanded to be 12, would the original optimal solution remains? If not, what it should be?

(20%)

3. Explain the following terminology in "queueing theory":

(3-a) lack of memory

(3-b) steady-state condition

(3-c) balk

(3-d) $M/G/s$ model

(3-e) Little's formula (有二種 formula, 要解釋 formula 的符號。)

(20%)

4. Consider a birth-death system with the following birth and death coefficients:

$$\lambda_k = (k+2)\lambda \qquad k = 0, 1, 2, \ldots$$

$$\mu_k = k\mu \qquad k = 1, 2, 3, \ldots$$

All other coefficients are zero.

(4-a) Draw the rate diagram (state-transition-rate diagram).

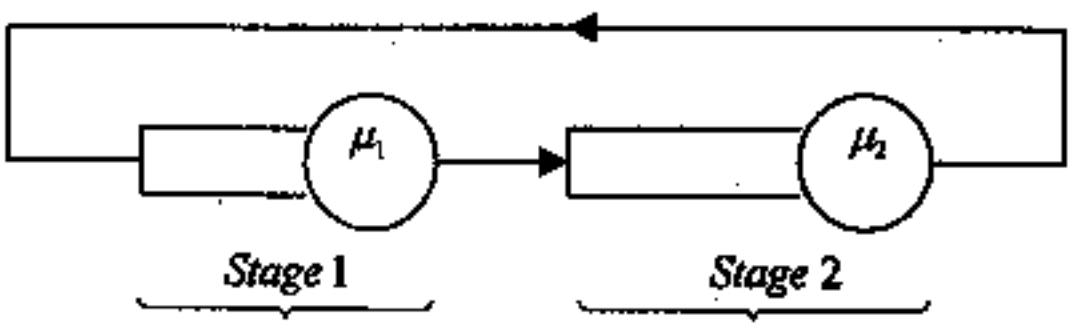
(4-b) Calculate P_0, P_1, P_2, P_3, and P_k for $k = 4, 5, \ldots$ (Be sure to express your answer explicitly in terms of λ, μ, and k only.)

(4-c) Find the average number of customers in the system.

(10%)

5. Consider a "cyclic queue" in which S customers circulate around through two queueing facilities as shown below. Both servers are of exponential type with rates μ_1 , μ_2 , respectively. Let

P_k = Prob{ k customers in stage 1 and S - k customers in stage 2 }



(5-a) Draw the rate diagram (state-transition-rate diagram).

(5-b) Write down the relationship among { P_k }. (不必解出 P_k。)