

八十八學年度 工業工程與工程管理系(所) 工工 組碩士班研究生招生考試

科目 線性代數 科號 2502 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (25%) True or False. Justify your answers.

a. All polynomials of the form $P(t)=a+bt^2$, where $a \in \mathbb{R}$, is a subspace of P^* .

b. A denotes an $m \times n$ matrix. $\text{Nul } A$ is the Kernel of the mapping $x \rightarrow Ax$.

c. If B is produced by multiplying row 2 of A by 3, then $\det B = 3\det A$.

d. If a partitioned matrix is $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, then $A^T = \begin{bmatrix} A_{11}^T & A_{21}^T \\ A_{12}^T & A_{22}^T \end{bmatrix}$.

e. If A is $m \times n$ and $\text{rank } A = m$, then the linear transformation $x \rightarrow Ax$ is one-to-one.

2. (15%) In P_2 , find the change-of-coordinates matrix from the basis

$B = \{1-3t^2, 2+t-5t^2, 1+2t\}$ to the standard basis. Then write t^2 as a linear combination of the polynomials in B .

3. (10%) Show that if A and B have the same rank, then they are similar.

4. (9%) (a) Let $A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$. Find

3% (1) The characteristic equation of A ;

3% (2) The eigenvalues of A ;

3% (3) The corresponding eigenvectors.

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5. (9%) (b) Let $A = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$. Find

3% (1) The characteristic equation of A ;

3% (2) The eigenvalues of A ;

3% (3) The corresponding eigenvectors.

6. (9%) Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}$.

Are A and B similar? Give the reason why.

7. (9%) Let $A = \begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix}$. Is A diagonalizable? Give the reason why.

8. (14%) Let $A = \begin{bmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{bmatrix}$, Find

(1) Eigenvalues of A ;

(2) Eigenvectors of A .

(3) Is it diagonalizable? If the answer is no, give the reason why.

If the answer is yes, give its diagonalized form and how to reach it.