國 立 清 華 大 學 命 題 紙

八十七學年度 工業工程 系（所） 工工甲 組碩士班研究生入學考試

統計學 科號 3301 3401 3501 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. (20 pts.) Suppose the lifetime of an electronic device has an exponential distribution given its density function as follows (unit: day):

$$f(x)=\begin{cases} 0.01e^{-x/\beta} & x>0 \\ 0 & elsewhere \end{cases}$$

(10 pts.) (a) Please find the probability that the lifetime of this device will be less than 50 days.

(10 pts.) (b) Suppose a system consisting of three of such devices in series. Please find the probability that the lifetime of this system will be less than 100 days.

2. (14 pts.) In Taiwan, the probability of a woman over 34 years old having a baby with Down Syndrome is 1/270. If the probability of a screening test correctly diagnosing a mother having a Down syndrome baby as positive is 0.60 (i.e., true positive) and the probability of incorrectly diagnosing mother not having a Down Syndrome baby as positive is 0.05 (i.e., false positive). Then, what is the probability that a mother over 34 years old diagnosed as having a Down syndrome baby is actually not having a Down Syndrome baby (i.e., posterior probability)?

3. (10 pts.) A chemical process converts lead (鉛) to gold. It is known that the process is normally distributed, with a standard deviation of 2.5g. How many samples must be taken to be 95% certain that an estimate of the mean process is within 1.5g of the true but unknown mean yield.

4. (10 pts.) A machine that fills cans with corn riblets (碎粒) should have a variance that does not exceed 0.03 (盎斯)^2 ; otherwise, the top will bulge (突起) and consumers will mistakenly think that the ingredients can cause botulism (肉類中毒). A sample of 12 cans yield a sample variance of 0.042 (盎斯)^2 . If $\alpha=0.05$, what is your conclusion ?

5. (10 pts.) A bakery has a line making Binkies, a big-selling junk food. Another line has just been installed, and the plant manager wants to know if the output of the new line is greater than that of the old line, as promised by the bakery equipment firm. Twelve days of data are selected at random from line 1 and 10 days of data are selected at random from line 2, with $\bar{x}_1=1124.25$ (箱) and $\bar{x}_2=1138.7$ (箱). It is known that $\sigma_1^2=52$ and $\sigma_2^2=60$. Test the appropriate hypotheses at $\alpha=0.05$, given that the outputs are normally distributed.

6. (21 pts.) Three players A, B, and C, standing in a line, play a game called passing hats. Player A passes hats to player B and player B passes hats to player C. The order can not be changed and each player try to receive the maximum amount of hats from the previous player. The number of hats received for each player is according to the following rule.

First, player A flips a fair die. If the die shows a number x, then player A receives x hats (automatically). Then, player B flips a fair die. If the die shows a number y, then player B receives y hats from player A if player A has enough hats to pass. Otherwise, player B receives all hats that player A can pass. Finally, player C flips a fair die. If the die shows a number z, then player C receives z hats from player B if player B has enough hats to pass. Otherwise, player C receives all hats that player B can pass.

(a) *Compute the expected number of hats player A receives.*

(b) Compute the expected number of hats player B receives.

(c) Compute the expected number of hats player C receives.

(d) Compute the variance of the hats player A receives.

(e) Compute the variance of the hats player B receives.

(f) Compute the variance of the hats player C receives.

(g) Interpret the expected numbers and variances for players A, B, and C for this game.

7. (15 pts.) Let $Y_1, Y_2, \ldots, Y_n$ be a random sample of size n. The sample mean $\bar{Y}$ is often used to estimate the population mean $E(Y)$. The measure of error $var(\bar{Y})$ is used to see how good of using $\bar{Y}$ to estimate $E(Y)$.

(a) Give two different unbiased estimators of $var(\bar{Y})$.

(b) Which estimater that you have listed in part (a) is better? Define "better" in your analysis.

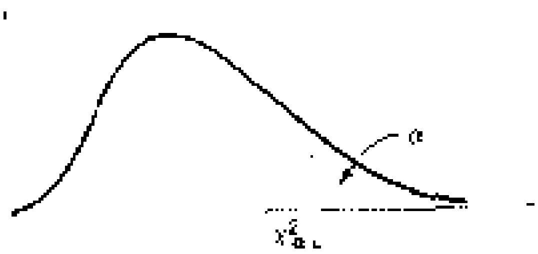
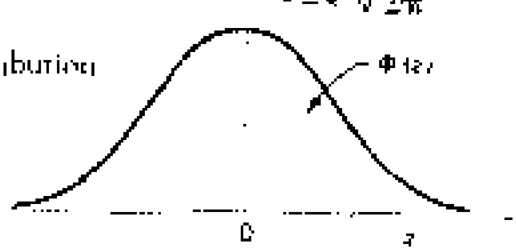


Table III Percentage Points $\chi^2_{\alpha,\nu}$ of the Chi-Square Distribution

ν \ α	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$$

Cumulative Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738