# 國立清華大學命題紙

八十六學年度 工業工程 系（所） 工工 組碩士班研究生入學考試

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1. **Yes or No.** If yes, give a brief proof, otherwise use an example to state why. (30%)

(1) A homogeneous system always has a solution.

(2) A nonhomogeneous system has a nontrivial solution if and only if the numbers of equations of the system equal the numbers of the variables.

(3) If a set $S = \{v_1, \ldots, v_p\}$ in $R^m$ contains the zero vector, then the set is linearly independent.

(4) If $v_1, \ldots, v_4$ are linearly independent vectors in $R^4$, then $\{v_1, v_2, v_3\}$ is also linearly independent.

(5) Let $T: R^n \longrightarrow R^m$ be a linear transformation and let $A$ be the standard matrix for $T$. Then $T$ maps $R^n$ onto $R^m$ if and only if the columns of $A$ span $R^m$.

(6) If an $m \times n$ matrix $A$ has $m$ pivot columns, then the equation $Ax = b$ has a unique solution for every $b$ in $R^m$.

(7) $A_{n\times n}$ is an invertible matrix iff the equation $Ax = 0$ has nontrivial solutions.

(8) We always can apply Cramer's Rule to find a solution of $Ax = b$.

(9) Given a $b \in R^m$, if the equation $Ax = b$ is consistent, then the column space of $A_{m\times n}$ is $R^m$.

(10) Any complex system can be simplified by investigating its corresponding eigensystem.

2. **Orthogonally diagonalize the following two matrices. (10%)**

(a)

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

(b)

$$\mathbf{B} = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

**Hint: Its characteristic equation is $-(\lambda - 7)^2(\lambda + 2) = 0$**

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3. Determine whether the linear systems are consistent. If they are consistent, are their solutions "unique"? Give the reason for your answer. (10%)

(a)

$$2x_2 + 2x_3 = 0$$
$$x_1 - 2x_4 = -3$$
$$x_3 + 3x_4 = -4$$
$$-2x_1 + 3x_2 + 2x_3 + x_4 = 4$$

(b)

$$-3x_2 - 6x_3 + 4x_4 = 9$$
$$-x_1 - 2x_2 - x_3 + 3x_4 = 1$$
$$-2x_1 - 3x_2 + 3x_4 = -1$$
$$x_1 + 4x_2 + 5x_3 - 9x_4 = -7$$

4. Describe the solutions of $\mathbf{Ax} = \mathbf{b}$, where

(a)

$$\mathbf{A} = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -2 & 4 \\ 6 & 1 & -8 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ -1 \\ -4 \end{bmatrix}$$

(b)

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -7 \\ -2 & -3 & 9 \\ 0 & -2 & 10 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Indicate specifically their solutions as the parametric vector form, such as $\mathbf{x} = \mathbf{p} + t\mathbf{v}$. This is to specify what are $\mathbf{p}$, $\mathbf{v}$, and $t$ respectively. (10%)

5.

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

The eigenvalues of the matrix A are *2* and *0*. Find $\mathbf{A}^{10}$. (5%)

6. Recall that an $n \times n$ matrix $N$ is *nilpotent* if $N^m = 0$ for some integer $m$.
   (6–a) Show that the eigenvalues of a *nilpotent* matrix $N$ are all zero. (10%)
   (6–b) Show conversely that if the eigenvalues of a matrix $N$ are all zero, then $N$ is *nilpotent*. (10%)

7. The number of an IC (Integrated Circuit) produced by a company for the last four-month period were as follows. Determine a **least square line** ( called the ***cost–volume formula*** ) and use it to predict the total cost for the next month (November) if production was planned to rise sharply to 1000 IC's. (15%)

| | July | Aug | Sept | Oct |
|---|---|---|---|---|
| **Number of IC produced** | 200 | 400 | 600 | 800 |
| **Total cost (in dollars)** | 820 | 1160 | 1430 | 1750 |