

八十五學年度 工業工程 系(所) 工程管理 組碩士班研究生入學考試

科目 微積分 科號 3602 共 4 頁第 1 頁 *請在試卷【答案卷】內作答

40%

1. 單一選擇題，每題四分，請依題號順序作答，將答案寫在「答案卷」上。

(1-1) $\int_0^1 \frac{x}{(x+1)^2(x^2+1)} dx =$

- (a) $3\pi/2$ (b) $(3\pi-2)/4$ (c) $(\pi-2)/8$ (d) $25/8$ (e) 8
 (f) none of the above.

(1-2) Let $f(x) = \begin{cases} e^{-(1/x^2)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. $f^{(2)}(0) =$

- (a) 2 (b) 1 (c) $2/3$ (d) 0 (e) -1
 (f) none of the above.

(1-3) Let $F(x) = (e^{3x} - 5x)^{(1/x)}$. $\lim_{x \rightarrow \infty} F(x) =$

- (a) e^{-1} (b) e^{-2} (c) e (d) e^2 (e) e^3
 (f) none of the above.

(1-4) $[y]$ is defined to be the greatest integer $\leq y$. $\int_0^{10} [x/3] dx =$

- (a) 15 (b) 12 (c) 10 (d) 8 (e) 1
 (f) none of the above.

八十五學年度 工業工程 系(所) 工程管理 組碩士班研究生入學考試

科目 微積分 科號 3602 共 4 頁第 2 頁 *請在試卷【答案卷】內作答

(1-5) The relative maximum point of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ is

- (a) $(-1, -2)$ (b) $(1, -2)$ (c) $(-1, 2)$ (d) $(1, 2)$ (e) $(0, 0)$
 (f) none of the above.

(1-6) F and G are two functions. We will write

$$F(x) = O(G(x))$$

if there exist a constant K and an integer N , such that

$$|F(x)| \leq K|G(x)| \quad \text{for all } x \geq N.$$

Let $[\]$ be the greatest-integer function and $f_1(x) = 1,000,000x^8 + x^5$, $f_2(x) = [x]!/10^{[x]}$, $f_3(x) = (1.1)^{[x]}$, $f_4(x) = x^{100000}$. Which of the following statements is true?

- (a) $f_2(x) = O(f_3(x))$ (b) $f_3(x) = O(f_4(x))$ (c) $f_4(x) = O(f_1(x))$
 (d) $f_4(x) = O(f_3(x))$ (e) $f_2(x) = O(f_4(x))$ (f) none of the above.

(1-7) $\lim_{x \rightarrow \infty} \left\{ \frac{\sqrt{x+1} + \sqrt{x+2} + \dots + \sqrt{2x-1}}{x^{(3/2)}} \right\} =$

- (a) 1 (b) 2 (c) $\sqrt{2} - 1/2$ (d) $\frac{2}{3}(2\sqrt{2} - 1)$ (e) $2\sqrt{2} - 1$
 (f) none of the above.

(1-8) The domain of convergence of the series: $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2^n (3n-1)}$ is

- (a) $-1 \leq x \leq 3$ (b) $-1 \leq x \leq 2$ (c) $-1 \leq x \leq 1$ (d) all $x \neq 0$
 (e) $x > 0$ (f) none of the above.

八十五學年度 工業工程系(所) 工程管理組碩士班研究生入學考試

科目 微積分 科號 3602 共 4 頁第 3 頁 *請在試卷【答案卷】內作答

- (1-9) The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3n-1}$ is (a) divergent. (b) absolutely convergent.
 (c) conditionally convergent. (d) convergent but it is not absolutely convergent.
 (e) a convergent alternating series. (f) none of the above.

(1-10) $\lim_{x \rightarrow \pi/2} \frac{e^{x^2/\pi} - e\pi/4 + \int_x^{\pi/2} e^{\sin t} dt}{1 + \cos 2x} =$

- (a) $\pi/2$ (b) π/e (c) e/π (d) $2e/\pi$ (e) $e/2\pi$
 (f) none of the above.

2. ^{10%} Let $f(x) = \begin{cases} 2x \sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0. \end{cases}$

- (2-1) Is $f(x)$ continuous on $(-\infty, \infty)$?
 (2-2) Does $f(x)$ have a derivative at $x = 0$?
 (You should give the reasons for your answers.)

八十五學年度 工業工程系(所) 工程管理組碩士班研究生入學考試

科目 微積分 科號 3602 共 4 頁第 4 頁 *請在試卷【答案卷】內作答

3.

6% (3-1).

$$\text{Let } f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 5 & \text{if } x = 3 \end{cases}$$

Is $f(x)$ is continuous on $(-\infty, \infty)$

If "Not", give the reason.

6% (3-2).

$$\text{Let } f(x) = \begin{cases} \frac{\tan(\sin x)}{\sin x} & \text{if } \sin x \neq 0 \\ 1 & \text{if } \sin x = 0 \end{cases}$$

Is $f(x)$ is continuous on $(-\infty, \infty)$

If "Not", give the reason.

10% (3-3). Find the solution of $xy'' = 2y'$

14% 4. Integrate $\int_0^1 x^m (1-x)^n dx$

14% 5. Integrate $\int_0^1 e^{-\frac{x^2}{2}} dx$