

Please show the details of your work for all questions. (two pages)

1. Test for exactness. If exact, solve. If not, find an integrating factor and then solve.

(a) $e^{-2\theta}(rdr - r^2d\theta) = 0$ (10%)

(b) $(2\cos y + 4x^2)dx - x\sin ydy = 0$ (10%)

2. Solve the initial value problem.

$$y'' + 4y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = -5 \quad (10\%)$$

3. Solve the ordinary differential equation.

$$y'' + 4y' + 4y = \cos 4t \quad (10\%)$$

4. Find the inverse Laplace transform of $f(t)$ of

$$F(s) = \frac{e^{-s}}{s^2 + \pi^2} + \frac{e^{-3s}}{(s+2)^2} \quad (10\%)$$

5. Find the inverse of the matrix

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}. \quad (10\%)$$

6. Find the Fourier series of the function $f(x)$,

which is assumed to have the period 2π ,

$$\text{and } f(x) = x^2 \quad (-\pi < x < \pi). \quad (10\%)$$

7. Using separating of variables to solve the following partial differential equations. (10%)

$$\frac{\partial^2 u(x, t)}{\partial t^2} = c^2 \frac{\partial^2 u(x, t)}{\partial x^2}, \quad u(0, t) = u(L, t) = 0,$$

$$u(x, 0) = f(x), \quad u_t(x, 0) = 0, \quad (0 \leq x \leq L).$$

8. Find $\text{curl } \vec{v}$, where $\vec{v} = \left[\ln(x^2 + y^2), 2 \tan^{-1}\left(\frac{y}{x}\right), 0 \right]$ is given with respect to right-handed Cartesian coordinates. (10%)

9. $\int_{-L}^L \cos \frac{m\pi x}{L} \cos \frac{n\pi x}{L} dx = ?$ [m, n are integers.] (10%)