

參考用

甲、選擇題：共 10 題，每題 4 分，共 40 分。請用大寫字母 A, B, C, D 或 E 答題，並將答案依題號順序寫在答案卷上。皆單選。

1. What is the maximum value $|f''(x)|$ for the function $f(x) = x^3(10 - 3x^2)$ on the closed interval $[0, 2]$?

- (A) 16 (B) 660 (C) $\frac{40}{\sqrt{3}}$ (D) $20\sqrt{3}$ (E) 360

2. If $\int_0^{x^2} f(t) dt = x \sin \pi x$, what is the value of $f(4)$?

- (A) 0 (B) -2π (C) 2π (D) $-\frac{\pi}{2}$ (E) $\frac{\pi}{2}$

3. What is the value of $\lim_{n \rightarrow \infty} \int_0^1 \frac{ny^{n-1}}{1+y} dy$?

- (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 0 (D) $+\infty$ (E) $-\infty$

4. Which of the following series *diverges*?

- (A) $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ (B) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$ (C) $\sum_{n=1}^{\infty} \frac{2n3^n}{n^n}$ (D) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2 + n} - n)$ (E) $\sum_{n=1}^{\infty} \sin \frac{1}{n^2}$

5. What is the value of the definite integral $\int_0^1 \frac{x^3 e^{x^2}}{(x^2 + 1)^2} dx$?

- (A) $\frac{2e-5}{8}$ (B) $\frac{e-2}{4}$ (C) $\frac{e-1}{2}$ (D) $e-1$ (E) None of the above

6. Which function has maclaurin series $\sum_{n=0}^{\infty} (-1)^n 2^n x^n$?

- (A) $\tan^{-1} 2x$ (B) $\ln(1+2x)$ (C) $\ln\left(\frac{1+x}{1-x}\right)$ (D) $\frac{1}{1+2x}$ (E) $\sin^{-1} 2x$

7. What is the area that lies inside the circle $r = 6$ and above the line $r = 3 \csc \theta$?

- (A) $12\pi - 9\sqrt{3}$ (B) $36\pi - 9\sqrt{3}$ (C) $12\pi + \sqrt{3}$ (D) $12\pi - \sqrt{3}$ (E) $36\pi - \sqrt{3}$

8. What is the value of the double integral $\int_0^{\ln 10} \int_{e^x}^{10} \frac{1}{\ln y} dy dx$?

- (A) $\ln 9$ (B) 9 (C) 10 (D) $\ln 10$ (E) None of the above

9. Which of the following curve is tangent to the surface $x^2 + y^2 - z = 1$ when $t = 1$?

- (A) $\mathbf{r}(t) = (2t-1)\mathbf{i} + \sqrt{t}\mathbf{j} + \sqrt{t}\mathbf{k}$ (B) $\mathbf{r}(t) = \sqrt{t}\mathbf{i} + (2t-1)\mathbf{j} + \sqrt{t}\mathbf{k}$
(C) $\mathbf{r}(t) = \sqrt{t}\mathbf{i} + \sqrt{t}\mathbf{j} + (2t-1)\mathbf{k}$ (D) $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + \sqrt{t}\mathbf{k}$ (E) None of the above

10. Let S be the surface parametrized by $\mathbf{r}(\theta, z) = 3 \sin 2\theta \mathbf{i} + 6 \sin^2 \theta \mathbf{j} + z \mathbf{k}$ where $0 \leq \theta \leq \pi$. Which of the following is equivalent to the surface differential $d\sigma$?

- (A) $\frac{1}{3} d\theta dz$ (B) $d\theta dz$ (C) $3 d\theta dz$ (D) $6 d\theta dz$ (E) $12 d\theta dz$

注意：背面有試題

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乙、填充題：共 5 題，每題 6 分，共 30 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. Find the limit: $\lim_{x \rightarrow 0^+} \left(\frac{\sin x}{x} \right)^{1/x^2}$. Answer : _____

2. Among all the points on the graph of paraboloid $z = 10 - x^2 - y^2$ that above the plane $x + 2y + 3z = 0$, find the point farthest from the plane. Answer : _____

3. Set up the integral with the order $dzdrd\theta$ for evaluating the triple integral of the function $F(x, y, z) = \sin(z^2)$ over the solid region D bounded below by the plane $z = 0$, laterally by the circular cylinder $(x - 1)^2 + y^2 = 1$, and above by the paraboloid $z = 2 - x^2 - y^2$.

Answer : _____ (Do not evaluate the integral.)

4. Use the transformation (chang of variables) $x = uv^{-1}$, $y = uv$ to evaluate

$$\iint_D (x^2 + y^2) dx dy$$

where $D = \{(x, y) : 1 \leq xy \leq 4, 1 \leq y/x \leq 4\}$. Answer : _____

5. At the point $(1, 2)$, the function $f(x, y)$ has a derivative of 2 in the direction toward $(2, 2)$ and a derivative of -2 in the direction toward $(1, 1)$. Find the derivative of f at $(1, 2)$ in the direction toward the point $(4, 6)$. Answer : _____

丙、計算、證明題：共 3 題，每題 10 分，共 30 分。須詳細寫出演算過程，否則不予計分。

1. Define $f(x) = \begin{cases} x^3 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$ Show that $f'(x)$ is continuous at $x = 0$.

2. Find the values $a \leq b$ such that $\int_a^b 1 - e^{(1-x^2)} dx$ has minimal value.

3. Let C be a simple closed smooth curve in the plane $x + 2y + 2z = 2$. (Orient C to be counterclockwise when view from above.) Show that

$$\oint_C 3y dx - 2z dy + x dz$$

depends only on the area of the region enclosed by C and not on the position or shape of C .