科目 應用數學 類組別 034, 033, 032

＊請在試卷答案卷（卡）內作答

Please show the details of your work for all questions.

1. Solve the following oridinary differential equations.

(1) $y' + 4x^2 y = (4x^2 - x)e^{-x^2/2}$ (10%)

(2) $\cos(x+y)dx + (3y^2 + 2y + \cos(x+y))dy = 0$ (10%)

(3) $y' = (4x^2 + y^2)/(xy)$ (10%)

(4) $2x^2 y'' + 4xy' + 5y = 0$ (10%)

(5) $y'' + 4y' + 4y = \cos 4t$ (10%)

2. Using separating of varables to slove the following partial differential equations.

(1) $\dfrac{\partial^2 u(x,t)}{\partial t^2} = c^2 \dfrac{\partial^2 u(x,t)}{\partial x^2}$, $u(0,t) = u(L,t) = 0$,

$u(x,0) = f(x)$, $u_t(x,0) = 0$, $(0 \le x \le L)$. (10%)

(2) $\dfrac{\partial u(x,t)}{\partial t} = c^2 \dfrac{\partial^2 u(x,t)}{\partial x^2}$, $u_x(0,t) = u_x(L,t) = 0$,

$u(x,0) = f(x)$, $(0 \le x \le L)$. (10%)

3. Find a general solution of the system

$$\begin{cases} y_1' = 2y_1 - 2y_2 \\ y_2' = 2y_1 + 2y_2 \end{cases} \quad (10\%).$$

4. Slove the initial value problem by the Laplace transform

$y'' + y' + 9y = 0$, $y(0) = 0.16$, $y'(0) = 0$. (10%)

5. Find the odd periodic extension of the function (half-range expansion)

$$f(x) = \begin{cases} \dfrac{2k}{L}x & \text{if} \quad 0 < x < \dfrac{L}{2} \\ \dfrac{2k}{L}(L-x) & \text{if} \quad \dfrac{L}{2} < x < L \end{cases} \quad (10\%).$$