

科目 工程數學 類組別 033,034,035 共 1 頁第 1 頁 *請在試卷答案卷(卡)內作答

10% 1. Solve $(y^2 + yx)dx - x^2 dy = 0$.

10% 2. Solve $3x^2 y'' + 6xy' + y = 0$.

10% 3. Solve $f(t) + \int_0^t (t-\tau)f(\tau)d\tau = t$ for $f(t)$.

10% 4. Determine whether the matrix $A = \begin{pmatrix} -5 & 9 \\ -6 & 10 \end{pmatrix}$ is diagonalizable. If so, find the matrix P that diagonalizes A and the diagonal matrix D such that $D = P^{-1}AP$.

10% 5. Evaluate $\oint_C zdx + xdy + ydz$, where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$. Orient C counterclockwise as viewed from above.

10% 6. Show that the set of functions $\{1, \cos nx\}$, $n = 1, 2, 3, \dots$ is orthogonal on the interval $[-\pi, \pi]$.

10% 7. Find the Fourier Series of $f(x) = \begin{cases} 1 & -1 < x < 0 \\ x & 0 \leq x < 1 \end{cases}$.

10% 8. Find product solutions of $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} + u$.

20% 9. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $0 < x < 1$, $t > 0$

Subject to: $u(0, t) = 0$, $u(1, t) = 0$, $t > 0$

$$u(x, 0) = 0, \quad \left. \frac{\partial u}{\partial t} \right|_{t=0} = \sin \pi x, \quad 0 < x < 1.$$