

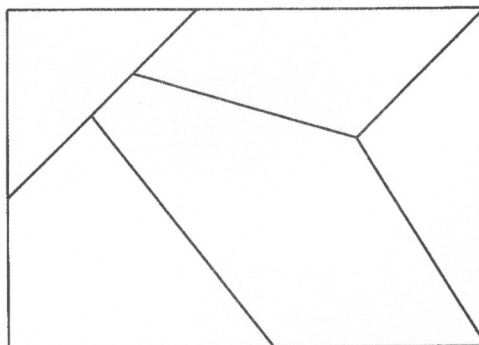
附件 2：台灣聯合大學系統 94 學年度學士班轉學生考試命題紙

科目 離散數學 類組別 D9 共 二 頁第 一 頁 \*請在試卷答案卷(卡)內作答

1. (10 points) The power set  $P(A)$  of set  $A$  is the set of all possible subsets of  $A$ . Justify the following statements by proving or disproving them:
  - (a)  $A, B$  are arbitrary sets, then  $P(A \cap B) = P(A) \cap P(B)$ .
  - (b) Given that  $A = \{a, \{a\}\}$ , then  $\{\{a\}\} \in P(A)$  and  $\{\{a\}\} \subseteq P(A)$ .
  
2. (15 points) Advanced compilers could re-arrange machine instructions so as to optimize performance and resource utilization. Consider a piece of assembly code listed as follows. An instruction can not be executed until all its operands are ready. For example,  $I_2$  **must be executed no earlier** than  $I_1$  is. The situation can be expressed as  $I_1 \rightarrow I_2$ . Let  $R = \{(I_a, I_b) \mid I_a \rightarrow I_b\}$  be a binary relation on the set of the five instructions.

$I_1$ : ADD  $R3 \leftarrow R1, R2$  ; add  $R1$  to  $R2$  and put the result in  $R3$   
 $I_2$ : SUB  $R4 \leftarrow R3, 1$  ; subtract 1 from  $R3$  and put the result in  $R4$   
 $I_3$ : MOV  $[100], R4$  ; put  $R4$ 's content into memory location 100  
 $I_4$ : MUL  $R1 \leftarrow R4, 5$  ; multiply  $R4$  by 5 and then put the result in  $R1$   
 $I_5$ : MOV  $[200], R1$  ; put the content of  $R1$  into memory location 200

- (a) List all items in  $R$ .
  - (b) Is  $R$  a partial order relation? Explain why or why not.
3. (10 points) An artist is coloring stained glass. Any two adjacent shapes on the glass must have different colors. What is the minimal number of colors the artist needs to paint the glass shown below?



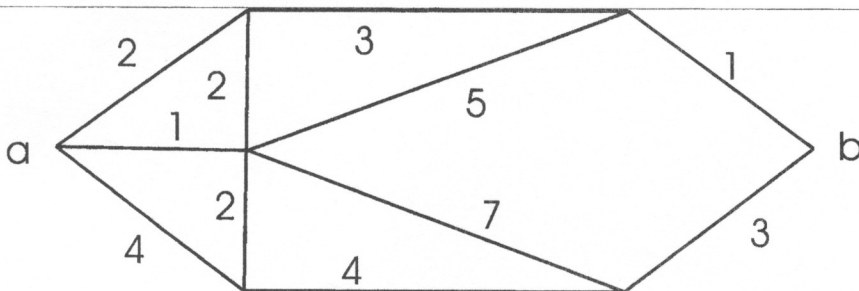
4. (15 points) Find the number of permutations of the six letters  $x, y, x, z, y, z$  so that no  $x$  appears in the first two elements, no  $y$  appears as the fourth element, and no  $z$  appears in the last two elements.

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5. (10 points) Is it true that a group with  $p$  elements (where  $p$  is a prime number) must be cyclic? Why or why not?

6. (10 points) Prove that if there is a vertex of degree  $k$  in a tree, there must be  $k$  vertices of degree 1 in that tree.

7. (15 points) Please write an algorithm for finding a shortest path from a vertex to another vertex. Use the following figure to explain your algorithm. Vertex  $a$  is starting vertex and vertex  $b$  is the final vertex.



Find a shortest path from  $a$  to  $b$ .

8. (15 points) Consider only positive integer written in decimal notation. Define a function  $f$  as follows:  $f$  moves the most significant digit of an integer to the least significant place. For instance,  $f(1234) = 2341$ ,  $f(564) = 645$ ,  $f(10032) = 321$ , etc. Find the least integer  $x$  such that  $f(x) = \frac{3x}{2}$ .