# 台灣聯合大學系統 93 學年度學士班轉學生考試命題紙

科目：應用數學　　類組別：C-5　　共 2 頁第 1 頁　　*請在試卷答案卷（卡）內作答

**1.** The nonhomogeneous equation $y' + p(x)y = r(x)$ has the general solution in the following form of an integral

$$y(x) = e^{-h(x)}\left[\int e^{h(x)} r(x)dx + c\right]$$

where $h(x) = \int p(x)dx$. Please show this solution formula! (20%)

**2.** The thickness of a bottomset bed at the foot of a delta can often be well approximated by the expression

$$t = t_0 \exp(-x/x_0) \qquad \text{(EQ.1)}$$

where $t$ is thickness, $x$ is distance from the bottomset bed start and $t_0$ and $x_0$ are constants.

**(a)** Imagine approximating this sedimentary bed in cross-section by a series of rectangles of thickness $t_i$ and width $\Delta x$ (Fig. 1). What is the area of each rectangle? (5%)

**(b)** Now write down an approximate sum for the cross-sectional area of the entire bottomset bed with a series of $N$ rectangles of equal width $\Delta x$ but different thickness. (5%)

**(c)** By considering the limiting case of an infinite number of infinitesimally wide rectangles, write down and evaluate an integral equation giving the total cross-sectional area. (10%)

**(d)** If the present-day rate of sediment supply is 10 $m^2$/year and $x_0$ = 5 km and $t_0$ = 1 m, estimate the time taken to form the bed assuming the sediment supply rate has not altered through time. (10%)

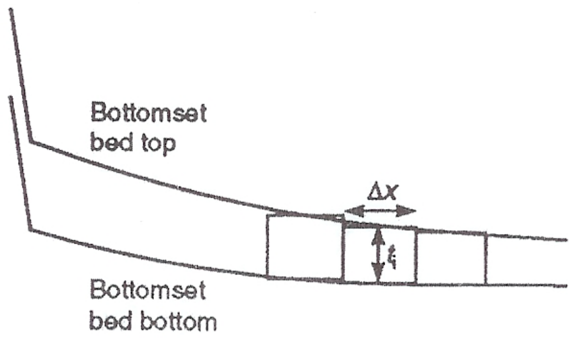


*Figure 1*

**3.** Considering that one-dimensional motion of a particle is governed by the following nonlinear differential equation:

$$\dot{x} = \sin x \qquad \text{(EQ.2)}$$

where $x$ is the position of this particle and $\dot{x}$ is the time-derivative of its position, i.e. the velocity.

**(a)** Suppose that $x(t = 0) = x_0$, please find the position function $x(t)$ for this particle. (10%) (Hint: $\int \csc u\,du = -\ln|\csc u + \cot u| + C$ and $\cos 2\theta = \cos^2\theta - \sin^2\theta$)

**(b)** Assuming $x_0 = \pi/4$, please plot the solution $x(t)$ qualitatively. What happens as $t \to \infty$? (10%)

**(c)** Now, let's consider another thinking way for (EQ.2). Please plot (EQ.2) in the so-called phase plane, that is the position as the abscissa and the velocity as the ordinate or the $x$-$\dot{x}$ plane, and draw the moving directions of this imaginary particle everywhere by arrows. (10%)

**(d)** For an arbitrary initial condition $x_0$, what is the behavior of $x(t)$ as $t \to \infty$? (10%) (Hint: Consider those points $x = n\pi$, $n$ is an integer)

**(e)** For any one-dimensional system $\dot{x} = f(x)$ as shown in Fig. 2, could you describe the general behavior of the particle motion qualitatively as the evolution of time? (10%)

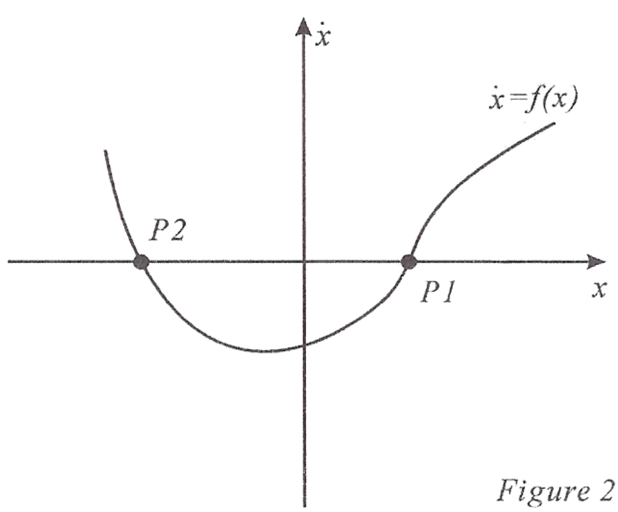


*Figure 2*