

一. 填充題. 共60分. (只需按標碼甲, 乙, 丙...等填出答案即可)

1. (7分)

Let $f(x)$ be a differentiable function on \mathbb{R} satisfying

$$f(x^2) = 1 + \int_0^{x^2} f(y)(1 - \tan y)dy$$

for all $x \in \mathbb{R}$. Then $f(\pi) =$ 甲.

2. (7分)

Let L be the line tangent to the polar curve $r(\theta) = \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}$ at $\theta = 0$. The equation of L in x and y is 乙.

3. (7分)

Evaluate the improper integral $\int_0^\infty \frac{dx}{(x+1)[x^2+(x+1)^2]}$ by transforming it into a definite integral of the form $\int_0^1 \frac{ay+b}{\alpha y^2+\beta y+\gamma} dy$ via an appropriate 1-1 onto differentiable function $[0, \infty) \xrightarrow{y=f(x)} [0, 1)$. Answer: 丙.

4. (7分)

Evaluate $\int_0^{\frac{\pi}{3}} \frac{1}{\sin x - \cos x - 1} dx =$ 丁.

5. (8分)

Let $p(x) = x^6 + 2x^5 - x + 1$. Find $\lim_{x \rightarrow \infty} \{(p(x))^{1/6} - x\} =$ 戊.

6. (8分)

Evaluate $\int_{\Omega} xy dx dy$, where Ω is the region in the first quadrant bounded by the curves: $x^2 + y^2 = 4, x^2 + y^2 = 9, x^2 - y^2 = 1, x^2 - y^2 = 4$. Answer: 己.

7. (8分)

Evaluate the line integral $\int_C (x^2 + 6xy - 2y^2)dx + (3x^2 - 4xy + 2y)dy$ along the path $C: y = \tan x$ from $x = 0$ to $x = \frac{\pi}{4}$. Answer: 庚.

8. (8分)

Find the volume of the solid T bounded above by the plane $z = 2y$ and below by the paraboloid $z = x^2 + y^2$. Answer: 辛.

二. 計算證明題. 共 40 分 (需寫出計算及證明過程, 否則不予計分)

1. (10分)

Find

$$\lim_{n \rightarrow \infty} \left(\sum_{k=1}^n \frac{n}{k^2 + n^2} \right).$$

2. (10分)

Let $0.a_1a_2a_3a_4\cdots$ be the decimal expansion of the rational number $\frac{5}{7}$. Let $b_k = a_{2k}, k = 1, 2, \dots$. The decimal $0.b_1b_2b_3b_4\cdots$ also represents a rational number $\frac{a}{b}$. Find $\frac{a}{b}$.

3. (10分)

Find the shortest distance from the point $(1, 2, 0)$ to the elliptic cone $z = \sqrt{x^2 + 2y^2}$.

4. (10分)

Evaluate the surface integral $\int \int_S (x^4 + y^4 + z^4) d\sigma$, where $d\sigma$ is the surface element and $S = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$.