

國立清華大學 命題紙

九十二學年度 原子科學系 系轉學生招生考試

科目 微積分 科號 0161 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

一、

1. 填充題：請將答案按字母順序寫在答案紙前八行。不要寫計算過程。違反規定者不予計分。(每格 8 分)

- (a) Let $D = \{(x, y) : x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}$, then $\iint_D \frac{2xy}{x^2+y^2} dx dy = \underline{(A)}$.
- (b) If $H(x) = 3x \int_4^{x^2} e^{-\sqrt{t}} dt$, then $H'(2) = \underline{(B)}$.
- (c) The circular helix defined by $\mathbf{r}(t) = (\cos t, \sin t, t)$ with $t \in [0, 4\pi]$ has arc length = (C).
- (d) Let f be a non-zero differentiable function, and $(f(x))^2 = 2 \int_0^x f(t) dt$, then $f(x) = \underline{(D)}$.
- (e) Which of the following limits exist? (E) (Don't evaluate the limits, just write the Greek alphabets.)

$$(\alpha) \lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}, \quad (\beta) \lim_{(x,y) \rightarrow (0,0)} \frac{xy}{|x|+|y|}, \quad (\gamma) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2-y^2}{x^2+y^2},$$

$$(\delta) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x-y)}{\cos(x+y)}, \quad (\epsilon) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(xy)}{x^2+y^2}.$$

- (f) Let $Q(t)$ be the solution of the initial value problem

$$\begin{cases} \frac{dQ}{dt} = 2Q(3-Q), \\ Q(0) = \frac{1}{2}, \end{cases}$$

then $\lim_{t \rightarrow \infty} Q(t) = \underline{(F)}$.

(g) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right) = \underline{(G)}$.

(h) $\int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx = \underline{(H)}$.

二、計算與證明：請詳細寫出每一推導步驟。

2. (11 points) Find the maximum and minimum values of $x+y^2z$ subject to the constraints $y^2+z^2=2$ and $z=x$.
3. (15 points) Suppose $f(x)$ is a function with derivative $f'(x) = \frac{x}{1+x^2}$. Show that for $a, b \in \mathbf{R}$

$$|f(b) - f(a)| \leq \frac{1}{2}|b-a|.$$

4. (10 points) Check the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{2}{1+e^n},$$

and give a proof.