國 立 清 華 大 學 命 題 紙

九十二學年度 計量財務金融學系 系轉學生招生考試

科目 微積分 科號 0143 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

一、

1. 填充題：請將答案按字母順序寫在答案紙前八行。不要寫計算過程。違反規定者不予計分。(每格 8 分)

(a) Let $D = \{(x,y) : x \geq 0, y \geq 0, x^2+y^2 \leq 1\}$, then $\iint_D \frac{2xy}{x^2+y^2} dxdy =$ (A).

(b) Suppose $f(t)$ is a continuous function and $F(x,y) = \int_{x-y}^{x+y} \sin(t^2)dt$, then $\frac{\partial F}{\partial y} =$ (B).

(c) The curve defined by $\mathbf{r}(t) = (\ln t, 2t, t^2)$ with $t \in [1, e]$ has arc length= (C).

(d) Let f be a non-zero differentiable function, and $(f(x))^2 = 2\int_0^x f(t)dt$, then $f(x) =$ (D).

(e) Which of the following limits exist? (E) (Don't evaluate the limits, just write the Greek alphabets.)

$$(\alpha) \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}, \quad (\beta) \lim_{(x,y)\to(0,0)} \frac{xy}{|x|+|y|}, \quad (\gamma) \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2},$$

$$(\delta) \lim_{(x,y)\to(0,0)} \frac{\sin(x-y)}{\cos(x+y)}, \quad (\epsilon) \lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x^2+y^2}.$$

(f) Let $Q(t)$ be the solution of the initial value problem

$$\begin{cases} \frac{dQ}{dt} = Q(2-Q), \\ Q(0) = \frac{1}{2}, \end{cases}$$

then $\lim_{t\to\infty} Q(t) =$ (F).

(g) $\lim_{x\to 0} x\lfloor \frac{1}{x} \rfloor =$ (G), where $\lfloor\cdot\rfloor$ is the Gauss step function, i.e. $\lfloor t \rfloor = n$, when $n \leq t < n+1$, $n \in \mathbf{Z}$ (the integers).

(h) $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x} dx =$ (H).

二、計算與證明：請詳細寫出每一推導步驟。

2. (11 points) Find the maximum and minimum values of $x+y^2z$ subject to the constraints $y^2+z^2=2$ and $z=x$.

3. (15 points) Suppose $f(x)$ is a function with derivative $f'(x) = \frac{x}{1+x^2}$. Show that for $a, b \in \mathbf{R}$

$$|f(b)-f(a)| \leq \frac{1}{2}|b-a|.$$

4. (10 points) Check the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{2}{1+e^n},$$

and give a proof.