

國立清華大學 命題紙

九十二學年度 經濟學系 系轉學生招生考試

科目 微積分 科號 0123 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

一、

1. 填充題：請將答案按字母順序寫在答案紙前八行。不要寫計算過程。違反規定者不予計分。(每格 8 分)

(a) $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx dy = \underline{(A)}.$

(b) $\lim_{x \rightarrow \infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}} = \underline{(B)}.$

(c) The circular helix defined by $\mathbf{r}(t) = (\sin t, \cos t, 2t)$ with $t \in [0, 4\pi]$ has arc length = (C).

(d) $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^2+1} dt = \underline{(D)}.$

(e) Which of the following limits exist? (E) (Don't evaluate the limits, just write the Greek alphabets.)

$$(\alpha) \lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}, \quad (\beta) \lim_{(x,y) \rightarrow (0,0)} \frac{xy}{|x| + |y|}, \quad (\gamma) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2},$$

$$(\delta) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x-y)}{\cos(x+y)}, \quad (\epsilon) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(xy)}{x^2 + y^2}.$$

(f) Let $Q(t)$ be the solution of the initial value problem

$$\begin{cases} \frac{dQ}{dt} = 3Q(2-Q), \\ Q(0) = \frac{1}{2}, \end{cases}$$

then $\lim_{t \rightarrow \infty} Q(t) = \underline{(F)}.$

(g) $\int_0^\infty x^2 e^{-x^2} dx = \underline{(G)}.$

(h) $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx = \underline{(H)}.$

二、計算與證明：請詳細寫出每一推導步驟。

2. (11 points) Find the maximum and minimum values of $x+y^2z$ subject to the constraints $y^2 + z^2 = 2$ and $z = x$.

3. (15 points) Suppose $f(x)$ is a function with derivative $f'(x) = \frac{x}{1+x^2}$. Show that for $a, b \in \mathbf{R}$,

$$|f(b) - f(a)| \leq \frac{1}{2}|b - a|.$$

4. (10 points) Show that the sequence $\{x_n\}_{n=1}^\infty$ and the series $\sum_{n=1}^\infty (x_{n+1} - x_n)$ both converge or diverge simultaneously.