國 立 清 華 大 學 命 題 紙

九十二學年度 化學工程學系 系轉學生招生考試

科目 微 積 分 科號 0063 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

I. 填充題（共六題，每題8分，請將答案依甲、乙、丙...次序作答，不需演算過程）

1. The Fibonacci sequence $\{a_n\}_{n=1}^{\infty}$ is defined as $a_1 = 1$, $a_2 = 2$, $a_{n+1} = a_n + a_{n-1}$ for $n \geq 2$. Then $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} =$ ___甲___.

2. Evaluate $\int_0^{\infty} e^{-x^2} dx =$ ___乙___.

3. Consider the arc $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$. Let S be the area of the surface obtained by rotating this arc about the x-axis. Then $S =$ ___丙___.

4. Let I be the **interval of convergence** of the series

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)(1-x)^n.$$

Then $I =$ ___丁___. (Note. Check the end points for convergence.)

5. Let m be the **absolute minimum** value of

$$f(x, y) = 2 + 2x + 2y - x^2 - y^2$$

on the closed triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$. Then $m =$ ___戊___.

6. Let L be the length of the arc

$$x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

Then $L =$ ___己___.

II. 計算與證明題（必須寫出演算證明過程）

1. (11%) Prove that the function $f(x) = (1+x)^{\frac{1}{x}}$ is strictly decreasing on the interval $(0, \infty)$.

2. (11%) Use the Maclaurin series for e^x to estimate

$$\int_0^{\frac{\pi}{2}} e^{-x^2}\,dx$$

with an error less than 10^{-2}.

3. (5%) (a) Sketch the graph of the solid D bounded by the surfaces

$$z = x^2 + 3y^2 \text{ and } z = 8 - x^2 - y^2.$$

(10%) (b) Find the volume of the solid D.

4. (15%) **Apply the method of Lagrange multipliers to find the point p which minimizes**

$$x^2 + y^2 + z^2$$

subject to the constraints $x + 2y + 3z = 6$ and $x + 3y + 9z = 9$. (Note. You should use the method of Lagrange multipliers only to get scores.)