

國 立 清 華 大 學 命 題 紙

九十二學年度 化學 系轉學生招生考試

科目 微 積 分 科號 0041 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

I. 填充題 (共六題, 每題 8 分, 請將答案依甲、乙、丙...次序作答, 不需演算過程)

1. $\lim_{n \rightarrow \infty} (1^n + 2^n + \cdots + 2003^n)^{1/n} =$ 甲

2. Given that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}.$$

Let $f(t) = \int_{-\infty}^{\infty} e^{-tx^2} dx$, $t > 0$. Then $f'(t) =$ 乙

3. Compute $\int_0^{\pi} \frac{dx}{1 + \sin x} =$ 丙

4. Let I be the interval of convergence of the series

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \right) (1-x)^n.$$

Then $I =$ 丁. (Note. Check the end points for convergence.)

5. Let m be the absolute minimum value of

$$f(x, y) = 2 + 2x + 2y - x^2 - y^2$$

on the closed triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, $y = 9 - x$. Then $m =$ 戊

6. Let L be the length of the arc

$$x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

Then $L =$ 己

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II. 計算與證明題（必須寫出演算證明過程）

1. (11%) Prove that the function $f(x) = (1+x)^{\frac{1}{2}}$ is strictly decreasing on the interval $(0, \infty)$.

2. (11%) Evaluate numerically the integral

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{x} dx$$

to an accuracy to two decimal places; that is, find a number I^* such that $|I - I^*| < 0.005$.

3. (5%) (a) Sketch the graph of the solid D bounded by the surfaces

$$z = x^2 + 3y^2 \text{ and } z = 8 - x^2 - y^2.$$

(10%) (b) Find the volume of the solid D .

4. (15%) Show that a positive constant t can satisfy

$$e^x > x^t \text{ for all } x > 0$$

if and only if $t < e$.