# 國 立 清 華 大 學 命 題 紙

九十二學年度 八系聯招 系轉學生招生考試

科目 微積分 科號 0003 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

## I. 填充題(共八題,每題7分,請將答案依甲.乙.丙...次序作答,不需演算過程)

1. If $\lim_{x\to 0} \dfrac{\sin 3x + a + bx + cx^3}{x^3} = -3$, then $a+b+c=$ 甲 .

2. $\int_0^1 \dfrac{\sqrt[4]{x}}{1+\sqrt{x}}dx =$ 乙 .

3. $\int_0^{\frac{\pi}{2}} |\cos 2x - \sin x| dx =$ 丙 .

4. Suppose $f(x)$ is a differentiable function on $(0,\infty)$ and satisfies

$$f(x^2) = \frac{1}{x^3}\int_4^{x^2} [3t^2 - f'(t)]dt.$$

Then $f'(4) =$ 丁 .

5. Let $V$ be the volume of the solid in the first octant that is inside the cylinder $x^2+y^2=1$ and bounded below by the $xy$-plane and above by the plane $3x+2y+6z=6$. Then $V=$ 戊 .

6. The smallest value of the function $f(x) = e^{-x}\sin x$ on the interval $[-\frac{\pi}{2}, \frac{3\pi}{2}]$ is 己 .

7. If $\sum_{n=0}^{\infty} a_n(x-1)^n$ is the power series expansion of $xe^x$, then $a_3 =$ 庚 .

8. Under the conditions $x>0, y>0, z>0$ and $x^2+y+z=25$, the maximum value of the product $xyz$ is 辛 .

## II. 計算與證明題(必須寫出演算證明過程)

(8分) 1. Does there exist a differentiable function $f : \mathbb{R} \to \mathbb{R}$ such that $f(0) = 0$ and $f(f(x)) = x^4 - x$ for all $x \in \mathbb{R}$? Give reasons for your answer.

(10分) 2. Let $a_1 = 1$ and, for $n \geq 2$, let $a_n = \sqrt{6 + a_{n-1}}$. Prove that $\lim_{n\to\infty} a_n$ exists and find the limit.

(12分) 3. Suppose $f(x)$ is a continuous, nonnegative function on $[0, 1]$ and, for $n = 0, 1, 2, \cdots$, let $a_n = \int_0^1 f(x)x^n dx$.

(a) Prove that $a_2 \leq a_1 \leq a_2 + \frac{a_0}{4}$.

(b) Prove that $\lim_{n\to\infty} a_n = 0$.

(14分) 4. (a) Compute the area of the region

$$D = \{(x, y)/4x^2 \leq y \leq (4x + 1)^2 + 3 \text{ and } y \leq 4\}.$$

(b) Let $(\bar{x}, \bar{y})$ be the centroid of $D$. Find $\bar{x}$.