國 立 清 華 大 學 命 題 紙

九十學年度 原子科學系 轉學生招生考試

科目 微積分 科號 161 共 1 頁 第 1 頁 *請在試卷【答案卷】內作答

I. 填充題（共五題，每題八分，請將答案依甲、乙、丙....次序作答，不需演算過程）

(1). Let $f(x) = \int_1^x t^{\frac{1}{3}}\,dt$, $x \in [1,5]$. Which of the following is true? Ans. 甲

a. $f(1) > 0$ b. $f(5) < 0$ c. $f(2) > f(4)$ d. $f(2) < f(4)$

(2). Find the limit $\lim_{n\to\infty}\left(\sum_{k=1}^{n} \frac{k^{10}}{n^{11}}\right)$. Ans. 乙

(3). Evaluate the integral $\int_8^{27} \frac{dx}{x - x^{1/3}}$. Ans. 丙

(4). Evaluate the integral $\int_0^1 \int_x^1 x e^{2y^3}\, dy dx$. Ans. 丁

(5). Find the area of the top half of the region inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = \cos\theta$. Ans. 戊

II. 計算與證明題（每題十二分，必須寫出演算證明過程）

(1). Let f be a C^1 function on $\mathbb{R}$. Verify that $z = f(\frac{y}{x})$ satisfies the equation $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$.

(2). Let f be a continuous real-valued function defined on $\mathbb{R}$. Using integration by parts, prove

$$\int_0^x \left(\int_0^t f(z)dz\right)dt = \int_0^x f(t)(x-t)dt.$$

(3). Let $\{a_n\}_{n=1}^{\infty}$ be a sequence with nonnegative terms. Suppose that $\sum_{n=1}^{\infty} a_n^2$ converges. Does $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converge? Prove or disprove it.

(4). Find $\int_D y^2 dA$, where D is the region bounded by the lines $x - 2y = 2$, $x - 2y = 5$, $2x + 3y = 1$ and $2x + 3y = 3$.

(5). Evaluate the line integral

$$\int_C (-y + e^x)dx + (x^3 + \sin y)dy,$$

where the curve $C = \{(x,y) \mid x^2 + y^2 = 1\}$ is traversed counterclockwise.