一. 填充題(共有十個空格，每一空格七分，請將答案依甲、乙、丙...次序寫出，不需演算過程)

1. The line tangent to the curve $y^4+y^3-xy+1=0$ at the point $(3,1)$ is __(甲)__.

2. $\lim_{n\to\infty} n\int_0^n e^{x^2-n^2}dx =$ __(乙)__.

3. The area of the region lying inside the graphs of the circles $x^2+y^2=1$ and $x^2+(y-1)^2=1$ is __(丙)__.

4. Let I be the subset of $\mathbb{R}$ consisting of all x such that $\sum_{n=1}^{\infty}\frac{2^n e^{nx}}{n^2}$ converges. Then $I =$ __(丁)__.

5. Let $f:[0,1]\to\mathbf{R}$ be a continuous function, and suppose that $\int_0^1 f(x)dx=\pi$ and $\int_0^1 xf(x)dx=\sqrt{3}$. Then
$\int_0^1\int_0^x f(x-y)dydx =$ __(戊)__;
$\iint_{x^2+y^2\le 1} x^2 f(x^2+y^2)dxdy =$ __(己)__.

6. The cylinder $x^2+y^2=1$ and the plane $x+z=1$ meet in an ellipse E. The parametric equations for the line tangent to E at the point $P(0,1,1)$ is __(庚)__.

7. Let Γ be the boundary of the region
$$\{(x,y)|\ 0\le x\le\pi \text{ and } 0\le y\le\sin x\},$$
traversed counterclockwise. Then the line integral $\oint_\Gamma ydx+\sin xdy =$ __(辛)__.

8. Let $f(x) = \int_1^x \sin(t^2)dt$. Then

$$\lim_{n\to\infty} n^2[f(\sqrt{\pi}+\frac{1}{n}) + f(\sqrt{\pi}-\frac{1}{n}) - 2f(\sqrt{\pi})] = \underline{\quad (壬) \quad}.$$

9. Let Ω be the region $\{(x,y)|\ 0 \le x \le 1 \text{ and } 0 \le y \le x\}$. Then

$$\iint_\Omega \frac{1}{(1+x^2+y^2)^{3/2}}dxdy = \underline{\quad (癸) \quad}.$$

二. 計算與證明題(共三題，合計三十分，必需寫出演算證明過程)

1. (9分) Does the series $\sum_{n=1}^{\infty}\left(\sin\frac{1}{2n} - \sin\frac{1}{2n+1}\right)$ converge? Give reasons for your answer.

2. (9分) Let $f : [0, 2\pi] \to \mathbb{R}$ be a continuously differentiable function. Show that

$$\left|\int_0^{2\pi} f(x)\cos nx dx\right| \le \frac{1}{n}\int_0^{2\pi} |f'(x)|dx$$

for any positive integer n, and then conclude that

$$\lim_{n\to\infty}\int_0^{2\pi} f(x)\cos nx dx = 0.$$

3. (12分) Find the length of the shortest chord(弦) that is normal to the parabola $y^2 = x$ at one end of the chord.