一、填充題(共有十個空格，每一空格七分，請將答案依甲、乙、丙...次序寫出，不需演算過程)

1. $\int_0^1 \ln(x+\sqrt{x^2+1})dx =$ ___(甲)___.

2. The line normal to the curve $x^4 + x^3y^3 + y^4 = 1$ at the point $(1,-1)$ is ___(乙)___.

3. The area of the region lying inside the graphs of the circles $x^2+y^2=1$ and $x^2+(y-1)^2=1$ is ___(丙)___.

4. $\lim_{n\to\infty} n\int_0^n e^{x^2-n^2}dx =$ ___(丁)___.

5. Let I be the subset of $\mathbb{R}$ consisting of all x such that $\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{x-1}{x}\right)^n$ converges. Then $I =$ ___(戊)___.

6. Let $f:[0,1]\to\mathbb{R}$ be a continuous function, and suppose that $\int_0^1 f(x)dx = \pi$ and $\int_0^1 xf(x)dx = \sqrt{3}$. Then
$\int_0^1\int_0^x f(x-y)dydx =$ ___(己)___;
$\iint_{x^2+y^2\le 1} x^2 f(x^2+y^2)dxdy =$ ___(庚)___.

7. Let Γ be the boundary of the region

$$\{(x,y) \mid 0\le x\le \pi \text{ and } 0\le y\le \sin x\},$$

traversed counterclockwise. Then the line integral
$\oint_\Gamma y\,dx + \sin x\,dy =$ ___(辛)___.

8. Let $f(x)=\int_1^x \sin(t^2)dt$. Then

$$\lim_{n\to\infty} n^2[f(\sqrt{\pi}+\frac{1}{n})+f(\sqrt{\pi}-\frac{1}{n})-2f(\sqrt{\pi})] = \underline{\quad(壬)\quad}.$$

9. Let Ω be the region $\{(x,y)|\ 0\le x\le 1 \text{ and } 0\le y\le x\}$. Then $\iint_\Omega \frac{1}{(1+x^2+y^2)^{3/2}}dxdy = \underline{\quad(癸)\quad}$.

二. 計算與證明題(共三題，合計三十分，必需寫出演算證明過程)

1. (8分) Does the series $\sum_{n=1}^{\infty}\left(\sin\frac{1}{2n}-\sin\frac{1}{2n+1}\right)$ converge? Give reasons for your answer.

2. (10分) The cylinder $x^2+y^2=1$ and the plane $x+z=1$ meet in an ellipse E.

   (a) Show that the length of the ellipse E is given by

   $$\int_0^{2\pi}\sqrt{1+\sin^2 t}\ \ dt.$$

   (b) Find parametric equations for the line tangent to E at the point $P(0,1,1)$.

3. (12分) Find the maximum and minimum values of the function $f(x,y)=x^2+4y^2-xy-y$ in the triangular region $\{(x,y)|\ 0\le y\le 1 \text{ and } 0\le x\le y\}$.