國立清華大學命題紙

八十八學年度轉學生入學考試

科目 微積分（一般） 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

I. 填充題 （共有十個空格，每一空格七分，請將答案依甲、乙、丙 ... 次序寫出，不需演算過程）

1. Let $y = \sqrt[3]{(x+1)\sqrt[3]{(x^2+1)\sqrt[3]{(x^3+1)}}}$. Then $\frac{dy}{dx}|_{x=0}$ = ___甲___.

2. Evaluate the following:

   (a) $\int_0^{\pi/4} \frac{\cos\theta}{\sqrt{2-\sin^2\theta}}\,d\theta =$ ___乙___.

   (b) $\lim_{n\to\infty} \frac{1}{n}\left[2^{\pi/n} + 2^{2\pi/n} + \cdots + 2^{n\pi/n}\right] =$ ___丙___.

   (c) $\sum_{n=0}^{\infty}\left(1-\frac{1}{n!}\right)\frac{1}{3^n} =$ ___丁___.

3. Let L be the tangent line to the curve $x^3 + y^3 + 3xy^2 = 1$ at the point (0,1). Then the area of the triangle formed by L and the coordinate axes is ___戊___.

4. The solution of the integral equation

$$f(x) = 1999 + \int_0^x f(t)\cos t\,dt$$

is given by $f(x) =$ ___己___.

5. The length of the parabolic spiral $r = \theta^2 \quad (\theta \geq 0)$

that lies inside the circle $r = 4$ is ___庚___.

6. Suppose the temperature distribution of a ball centered at the origin is

$$T(x,y,z) = \frac{100}{1+x^2+y^2+z^2}, \qquad x^2+y^2+z^2 \leq 20.$$

Then the direction (which is a unit vector) of greatest

increase of temperature at the point (1,2,3) is ___辛___.

7. Suppose the partial derivatives of $f(x,y)$ at the point of the semicircle $x = \cos t$, $y = \sin t \quad (0 \leq t \leq \pi)$ are

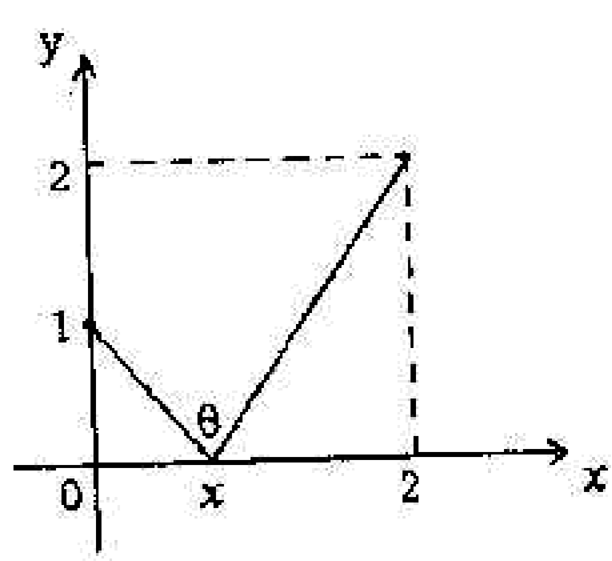
$$\frac{\partial f}{\partial x} = x - 2y \qquad \text{and} \qquad \frac{\partial f}{\partial y} = y - 2x.$$

Then on the semicircle, f has the maximum value at

$(x,y) =$ ___壬___.

8. The value of x that maximizes the angle θ in the figure

below is given by $x =$ ___癸___.



II. 計算與證明題（共有兩大題，每大題 15 分，必需寫出演算證明過程）

9. (15%)
Let $\Omega = \{(x,y) | 0 \le y \le 1, x \ge y \text{ and } x^2 - y^2 \le 1\}$.
(a) Sketch the region Ω.
(b) Evaluate the double integral

$$\iint_\Omega xy \sin(x^2 - y^2)\, dx dy.$$

10. (15%)
Suppose $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that

$$|a_n - n^2| \le \ln n \quad \text{for all } n.$$

(a) Does $\lim\limits_{n\to\infty} \dfrac{a_n}{n^2}$ exist? If so, find its value.

(b) Does $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{a_n}$ converge? Justify your answer.